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Solving Arithmetic Standard Word Problems Involving Representations and/or Language for Number Sense in Primary School:

A Literature Review

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Abstract
Solving arithmetic word problems (AWP), tasks for number sense have been identified as difficult in mathematics for learners since they require identifying, interpreting, and concluding words to arithmetic number notations, which is more difficult for primary school learners. The learner's lack of procedural knowledge leads to incorrect computation and incorrect answers. The difficulty stems from the teachers' inability to prepare linguistic, structural, and syntactic activities and tasks that help learners interpret and make sense of language and representational aspects to understand number notations. The study sought to examine a literature review on learning for number sense in primary school using arithmetic word problems involving representations and/or language. This study reviewed the literature on scientific articles published on linguistic and representational aspects of learning word problems in primary schools. After a manual screening process, 4 out of 52 articles were relevant to the study. The findings revealed that the reviewed papers in this study showed that learners in primary schools struggled to represent arithmetic word problems graphically and symbolically and eventually solve the problems. In conclusion, learners need to be guided in linking representations, symbols, and language when solving arithmetic word problems to help learners make sense of numbers.

Keywords
Language, Learning, Number sense, Representations, Word problems

INTRODUCTION

Learners experience difficulties interpreting arithmetic word problems (AWP) to number notation tasks and calculating them to get a correct answer in primary schools globally and in South Africa. Due to second language learning of numbers, some authors claim that difficulties with word problems on linguistic patterns, numeric complexity, logical structure complexity, and syntactic manipulation (Mandal et al., 2022; Hayashi et al., 2021; Ayala-Altamirano et al., 2022; Capone et al., 2021; Kwok et al., 2022; (Darockzy et al., 2015). Few authors identified that humans from birth possess an inborn number sense that needs to be developed (Graven et al., 2013). Most challenges are experiences with the way number computation is presented in schools; Sousa, (2015) identified that learners struggle to use operations, memorise multiple facts, and quickly get answers. Hence, they lack number sense.

Studies that use symbolic, verbal, and pictorial representations in the foundation phase (Grades 1-3) when solving AWP (Nasrun et al., 2023). Whereas in the intermediate phase (grades 4-6), AWP are written verbally and require learners to solve them using either symbolic (Rochat & Callaghan, 2005) and/or pictorial representations (Anwar & Rahmawati, 2017). To my knowledge, limited studies in the intermediate phase focus on the three pictorial, symbolic, and verbal representations when presenting and solving AWP. This study advocates for the use of verbal representation to present the situation. Learners must draw the situation to make a mental image, write a symbolic representation, and compare it with the problem situation. Once the learner is satisfied with the evaluation of the two situations, then, calculate the standard word problem developed to get to a correct answer. This study reviewed the literature on how arithmetic standard word problems involving representations and/or language are solved to assess learners' number sense in primary schools.

LITERATURE REVIEW

Learning arithmetic word problems

Given the importance of word problems involving representations and language in making sense of numbers in an educational context. There is no one definition of what learning is. However, some studies seem to agree that learning builds and flows from experience, and this happens when the learners' experience is engaged (Boud et al., 1993; Wrenn & Wrenn, 2009). Therefore, learning with understanding is essential to develop expertise since new learning is made easier, for example, through support transfer (National Research Council, 2000). In other words, if learners are not introduced to AWP to have experience with word problem tasks, they may not interpret, develop number notations, and solve them to get the correct answer.

Learning word problems is enhanced when teachers' pay special attention to learners' interpretation and provide scaffolding where necessary (National Research Council, 2000). The challenging part is that "mathematics word problems are transparently different from calculations because word problems are presented linguistically, challenging learners to read and interpret the problem, represent the semantic structure of the problem, and choose a solution strategy" (Schumacher & Fuchs, 2012, p. 608). Without a clear conceptualisation for AWP, the feedback provided by learners may be flawed or incorrect. Rossiter (2023, p. 35) that when "conceptualizing feedback as a dialogue places even greater importance on a successful interpretation of the message as a crucial step leading to the uptake of feedback". In this view, learning word problems with limited knowledge of AWP may struggle to understand the linguistic and semantic structure of the problem. This may result in a lack of AWP knowledge essential to solving the number notation.

Solving arithmetic word problems

There are different definitions of AWP. For this study, "Arithmetic word problems are usually written text about a problem, and it can be solved combining basic operations (addition, subtraction, multiplication, division) with numbers mentioned in the text which is directed towards primary school learners (Roy & Roth, 2016). Documented Difficulties in AWP stem from being unable to understand the written text about the problem, especially by second-language speakers. However, with adequate scaffolding and learner participation, it is possible to assist learners to understand plan and construct solutions to their problems (Dewi et al., 2023). Moreover, learning the material and methods that make sense of the text appropriate for improving learners' problem-solving competency is needed (Assapun & Thummaphan, 2023).

Some studies focused on transforming AWP from words into numerical computation (Hickendorff, 2021; Jan & Rodriguez, 2012) graphical illustrations and both numerical computation and graphical illustrations (Ayala-Altamirano et al., 2022; Sandwidi, 2018; Fatmanissa & Kusnandi, 2017).

Arithmetic standard word problems

Throughout the years, word problems have been part of school mathematics to accomplish a few goals and offer the practice of mathematical modelling and applied to problem-solve that learners apply the purpose of word problems (Verschaffel et al., 2020). There are different definitions of word problems; for this study word problems include a verbal description of an arithmetic standard word problems involving number computation have been difficult for learners in schools for many years. The difficulties stem from how learners interpret and make sense of AWP. The interpretation of word problems requires some form of linguistic and structural training in working with numbers. Number sense training in context may help address learners' deficits in arithmetic skills when solving word problems that may require working memory training (Kuhn & Holling, 2014).

In primary schools, AWP is presented in two forms, common sense, and the real world. Common sense word problems involve the use of common sense and common knowledge but not in-depth or specialist knowledge, which is essential for this study (Davis, 2023). At the same time, a real-world problem is a problem that uses nonmathematical knowledge to an extent to get to the solution (Davis, 2023). In AWP, learners collect basic linguistic knowledge and familiarity with basic operations (addition, subtraction, multiplication, and division) to make sense of numbers (Mandal & Naskar, 2019). Learning numbers sense requires learners to understand the AWP in the form of linguistic and mathematical structures and minimise complexity in computation.

Linguistic complexity

The language of learning and teaching AWP in some countries including South Africa, is the second language, making the explanations to learners' complex. Pallotti (2014) identified two terms where the word complexity is used: independent and dependent variables. The independent variable looks at the communicative tasks and identifies features that make them less or more complex (Pallotti, 2014), whereas the dependent variable describes the linguistic production of aspects. Studies that looked at the independent variable of complexity include Strohmaier et al. (2021), who identified "Linguistic factors to include syntactic and semantic complexity, text length, the presence of distractors or redundancies, and the relationship between language and mathematical terms" (p. 93). The meaning-making process by second language learners tends to be complex, requiring them to be able to represent number notation in the form of language, object, and symbol or combine all of them in the same activity. Learners may struggle to interpret the AWP, especially second-language speakers, without simplifying the mathematical language to mathematical symbolic notation.

Numerical complexity

Once the AWP is converted to arithmetic notation, learners are expected to compute it and get a correct answer. However, numerical complexity contributes to the difficulty in solving AWP. The complexity of numeric expressions is identified, simplified, and included in complexity prediction research (North et al., 2023).

Interpretation of word problems and logical structure complexity

Learners' logical reasoning of number computation is essential to develop the learners' logical structure of number concepts. Therefore, Wang et al. (2023) assert that the logical structures in text data can be uncovered by logical reasoning, which can perform conclusions to work out the answer. In addition, Tshesane & Venkat (2023), when citing Polotskaia's (2017), went further to categorise the interpretation process that is relevant to primary school learners working with AWP into three: interpretation of the problem, calculation of the unknown quantity, and evaluation of the numerical solution. Firstly, making sense of the problem and model development for sense-making and deriving a standard form with appropriate operations from the model Tshesane & Venkat (2023). Secondly, the unknown quantity is calculated by transforming the number sentence to be calculated to an answer Tshesane & Venkat (2023). Lastly, comparing the initial sense made from the problem and the sense made from the calculated solution was evaluated by Tshesane & Venkat (2023), and how the learners interpreted the AWP.

Hence, AWP is presented in textual form, and learners are required to interpret the text form, write it in numerical form, and later calculate it to get a correct answer. This process requires learners to use common-sense knowledge and real-world knowledge to make sense of the problem and calculate it. Learners who do not possess common sense (relational knowledge) and real-world knowledge may struggle to create a logical structure from AWP.

Polotskaia and Savard (2020) identified the Interpretation of mathematics problems and categorised it into two: the interpretation we give to the problem and mathematical reasoning relies on the flexibility of the interpretation. In addition, Dewi et al. (2023) state that interpretation helps to conclude where the meaning of the question is categorised and understood. Learners struggling to interpret the problem may struggle to be flexible and link the AWP to mathematical reasoning. Whereas learners who can interpret the AWP and can flexibly reason mathematically do not mean that they can calculate and get a correct answer.

Complexity in linguistic and mathematical syntactic

Concerning the relationship between English and Mathematics for second language speakers in general, attention to the learners' lack of knowledge can be reversed to look at issues of transmitting language knowledge to number notation in mathematics. Brannon (2005, p. 3178) asserts that "in light of the demonstrated independence of linguistic and mathematical syntax in the mature human, the syntax of mathematics may be evolutionarily or developmentally derived from the syntax of a language or possibly even vice versa". Brannon (2005) further states that though mathematics and language share the computational recursion principle, they are anatomically distinct according to their syntaxes. This suggests that interpreting AWP to number notation may present some challenges to primary school learners who struggle to connect the two. However, syntactic structures in a text can be more or less complex depending on the number of constituents and combinations they may take (Pallotti, 2015, p. 123).

A text showing a variety of regular syntactic patterns would be considered more complex than one with all sentences and clauses following one or two patterns or where no pattern is discernible at all (Pallotti, 2015). The teacher's choice of text is important to avoid complexity in AWP tasks in primary school.

Representations in solving arithmetic word problems in primary schools

The representations in solving AWP included pictorial, symbolic, and verbal-written representations (Nasrun et al., 2023). Elia (2020) identified that different semiotic characteristics of pictorial representations are often used as pictures involving illustrated groups of objects, numerals, or both. Pictorial representations include images, graphs, diagrams, and interrelated activities (Anwar & Rahmawati, 2017). Elia (2020) further states that pictures with different semiotic characteristics are likely to impact learners' interpretations of context and solutions to problems. Symbolic representations include operations, connection signs, numbers, algebraic symbols, and interconnected actions (Anwar & Rahmawati, 2017). Whereas verbal representations are words expressed in written and oral form (Rahmah et al., 2019). All three representations play an important role in taking a complete picture of the different stages of AWP and making sense of numbers.

CONCEPTUAL FRAMEWORK

The teaching philosophy of Payne and Rathmell's triangle (1975) has three aspects: language, representations, and symbols. Mathematical symbols are classified within the mathematical representations. Hence, there may be some connections between them. This study looks at mathematical symbols and classifies them into three symbolic development, graphical symbols, and pictorial symbols in primary school mathematics learning based on Rochat & Callaghan, 2005). Literature through language and play is demonstrated early in learners' development to dominate symbolic development (Rochat & Callaghan, 2005). Once learners are in the development stage, graphical symbols and intentional representations are not accidental things that resemble other things (Rochat & Callaghan, 2005). Moreover, the concrete representations in use and in learning require the pictorial symbols to form a special class of artifacts that are communicative in their function and deliberate in their making. In this view, external representations are usually symbolic in nature and are used in various learning, usually in written and spoken language (Sanwidi, 2018).

The language resides exclusively on the symbols' mastery in spoken language and an underdeveloped amount of analogy between the form of a sign and imitative words (Planner & Sterelny (2021). In most cases, a manifestation of word problems is presented in written language form. However, the problem solver amalgamates the written information into relevant situations or mental representations (Daroczy et al., 2015). Moreover, word problems are difficult and complex problems learners encounter in their primary mathematical development (Daroczy et al., 2015).

RESEARCH QUESTIONS

How do primary school learners solve arithmetic standard word problems involving representations and/or language for number sense in primary school?

MATERIALS AND METHODS

This qualitative research study used the analysis of the documents to review literature from the recommended articles on Google Scholar alerts on word problems sent to my Gmail account. The ethical approval was received from the University of the Free State, UFS-HSD2023/1531**.** To screen the relevant literature, I used articles that respond to language and/or representations in solving number problems on word problems in primary schools. The foundation phases grades 1-3 word problems were omitted in this study since the literature review focused on grades 4-7 learners since they use the home language and language of learning and teaching (English) in solving number tasks in mathematics. The literature review search was carried out on received emails from Google Scholar alerts in the period between the 9th of April 2023 and the 2nd of July 2023. Each manuscript's research title and abstract were copied and pasted into a Word document for easy access and to identify the identified codes for the study. The word problem articles were saved from my email account with Google Scholar alert for word problem articles. During the screening process, a careful reading and identification of the focus of a total of 52 abstracts from Google Scholar alerts on word problems was completed. Manuscripts were categorised into five. Firstly, 12 focused on word problems involving preservice and in-service teachers' instruction in mathematics. Secondly, 30 articles dealt with word problems in different mathematics topics. Thirdly, 03 articles looked at non-standard word problems involving numbers. Fourth, 03 articles focused on word problems involving learners with mathematics difficulties. Fifth, 04 research articles focused on the study, and after reading the full papers. They were categorised into languages = 1, symbols = 1, and representation = 2.
The inclusion criteria were used as follows:

- Research title.
- Focus on word problems in primary schools,
- Documents analysis, and
- Interpretation using Representations, symbols, and language

The inclusion criteria helped the author zoom in on the four articles analysed for this study. Data from the four research articles were analysed using Yin's (2016) five-stage analysis framework: compiling, disassembling, reassembling, interpreting, and concluding. Firstly, compiling is the critical stage before the data is segmented for analysis. The author had an opportunity to have a holistic view of the data (Creswell & Creswell, 2018). Screening helped me to look at the data holistically and compile it accordingly. Secondly, in disassembling data from the eight research articles were torn apart (screened to focus on number sense, language, representations, and a combination of both) groups (Castleberry & Nolen, 2018). Thirdly, in reassembling the grouped data, it is coded and categorised to find one emergent theme (learning number sense in primary school AWP involving representations and/or language) to answer the research questions (Yin, 2016). Fourthly, in the Interpreting stage, the emergent themes identified are described through the patterns found in the data to make conclusions (Castleberry and Nolen, 2018). Lastly, in the concluding stage, meaningful themes related to the conclusions to answer the research questions are merged (Yin, 2016). The results section addresses the interpreting and concluding stages with the support of conducted studies.

RESULTS

The emergent theme from the data analysis is presented under the results section and supported by the reviewed literature. The second last stage of Lin (2016), the analysis framework Interpreting and concluding helped to present the emergent theme in this section.

Solving arithmetic standard word problems involving representations and/or language for number sense in primary school is reviewed in the literature

Language and numerical complexities in solving arithmetic word problems

One research manuscript used language translated to arithmetic notation, the difficulties identified, strategies used to solve the word problems, and their findings. The two studies first presented the word problem in English written text to second-language English speakers. In contrast, one study presented a word problem in the Indonesian language and translated it into the English language. For word problems presented in English, some examples are presented below.

Manuscript 1-word problem: *"In total, 684 contestants from different countries participated in a cycling race. During the race, 248 contestants dropped out. How many contestants reached the finish?*

In manuscript 1, the word problem is presented in English for second-language learners, and they were required to represent the word problem in the arithmetic notation for one-step and two-step solutions in standard and nonstandard word problem types. For this study, nonstandard word problems were not considered. The study found that numerous processes are required to construct a situational model of the task when translating text into mathematical notation (Hickendorff, 2021). This suggests that focusing on standard word problems is aligned with the primary school curriculum in several countries worldwide. The study by Hickendorff (2021) further states that as learners progress at the school level, they can overcome the challenge of interpreting word problems into mathematical notations. In my view, making sense of standard word problems in a second language requires learners need to contextualise the problem into a familiar number computational structure.

Hickendorff (2021) found a non-significant advantage of word problems towards symbolic problems in Grade 5. Moreover, the two-step problems were not more complex than the one-step problem.

Representations and symbolic complexities in solving arithmetic word problems

The study presented one manuscript concerning representations and symbolic complexities though there are two learners' responses responding to two-word problem tasks.

Representational complexity has rarely been considered in solving AWP without including the mathematics notation due to the different ways word problems can be represented. A total of two-word problem tasks from one manuscript were presented one after the other and were written in the Indonesian language. See manuscript 2 first word problem below presented in Indonesian language and translated into the English language.

"Dalam perlombaan atletik, Jim berada empat meter di depan Tom dan Peter berjarak 3 m di belakang Jim. Seberapa jauh Peter di depan Tom?" *(In an athletics event, Jim is four meters ahead of Tom and Peter is 3m behind Jim. How far ahead of Peter is Tom?).*

The word problem statement above expected learners to write a graphical notation of the word problem task. The learner, Excel may not have struggled to make sense of the task written in the home language to get a correct answer. However, However, the learner's response in figure 1 below shows that the learner got an incorrect answer.

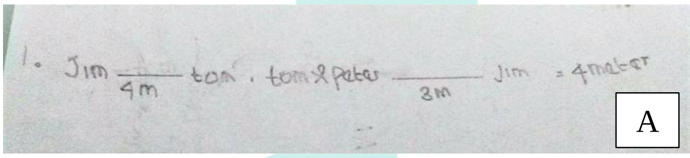

Fig. 1 Extract from Excel Work Results of Sanwidi representation of word problem question 1 task (2018, p. 150)

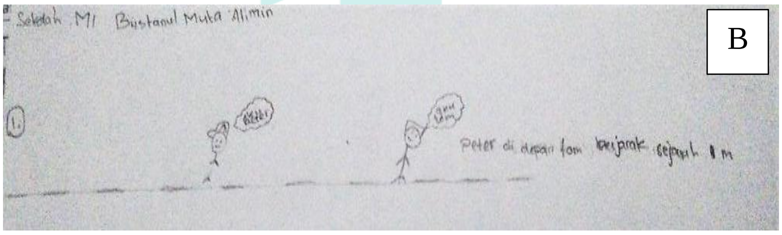

Fig. 2 Extract from Ihsan Work Results of Sanwidi representation of word problem question 1 task (2018, p. 150)

From question 1 of the task, the two responses above, Excel in figure 1 got an incorrect answer, and Ihsan in figure 2 got the correct answer. This suggests that though both learners interpreted from the Indonesian language to a graphical representation, one struggled when the other could represent the word problem correctly. This study seems to agree with the finding by Njagi (2015), who draws our attention to the fact that to create mental representations of word problems and to graphical representation, learners are unable to create mental representations. Limited research is conducted on how word problems presented in the home language help learners graphically represent word problems to make sense of numbers. One would expect learners to have a mental image of the problem and represent it graphically. When concrete objects do not exist, symbols are used (Knight, Goldsworthy, Visser & Hall, 2023) to help learners to reason abstractly when working with AWP. Therefore, combining representations and language to make sense of numbers may have helped the learners to visually see the difference between their responses and the correct answer. However, some authors may focus on the structural elements of the graphical representation as a challenge to learners.

In this section, I present question 2 in the same article with word problems presented in Indonesian and English. Learners were expected to interpret the task and represent it numerically and graphically. See the manuscript's second-word problem task response from Excel and Ihsan below.

"**Nindi berjalan kearah utara dari rumahnya menuju kekota A dengan jarak 66 Km, lalu dia pergi kekota B dengan jarak 13 Km kearah timur. Beberapa saat kemudian dia disuruh ibunya pergi kekota C sejauh 66 Km kearah selatan. Oleh ibunya dia disuruh untuk segera pulang kerumah, berapa jarak rumah si Nindi dari kota C?"** *(Nindi walked north from his house to town A with a distance of 66km, then he went to town B with a distance of*

13kmto east. A few moments later, he was told by his mother to go to city C as far as 66km to the south. His mother told him to go home immediately. How far is Nindi's house from city C?)
Excel used a symbolic approach while Sanwidi used blocks to solve a given standard word problem. Please see figures 3 and 4 below.

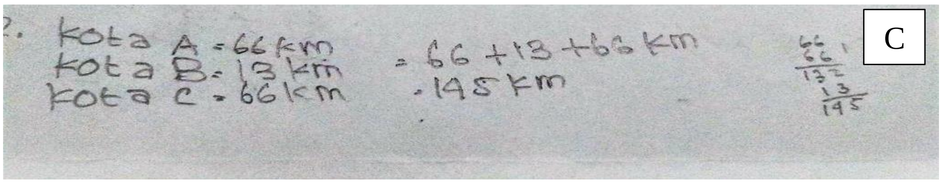

Fig. 3 Excel work results - Extract from Ihsan Work Results of Sanwidi representation of word problem question 2 task (2018, p. 150)

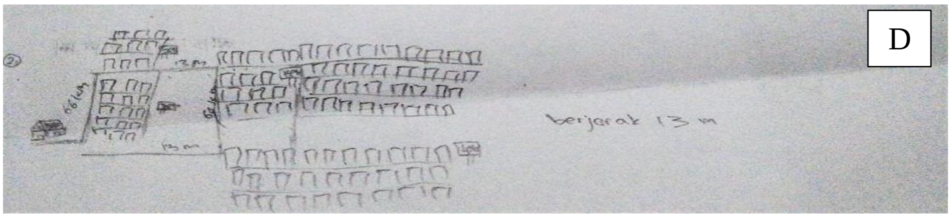

Fig. 4 Extract from Ihsan Work Results of Sanwidi representation of word problem question 2 task (2018, p. 150)

Excel in Figure 3 and Sanwidi in Figure 4 got the incorrect answers. Sanwidi solved the problem using blocks to represent a problem graphically. While Excel added the distances symbolically to get an incorrect answer. However, Sanwidi seems to have made a careless mistake and wrote 13m instead of 13km. That does not justify the wrong answer, but from the graphical representation, you may identify what may lead to an incorrect answer. This suggests that Sanwidi viewed the kilometers as meters, and that led to an incorrect answer. However, the working memory is accountable for the word problem differences in individuals, and it has the highest result in solving word problems than counterfeit mathematics skills (Ziadat, 2022, citing Friedman et al., 2018). Without a trained working memory in word problem tasks involving representations and language, learners may struggle to make sense of numbers when presented in their home language.

Language and representation complexities in solving arithmetic word problems
The two manuscripts had language and representation complexities in solving AWP. In both cases, the learners were given word problems in the English language and were to create images of how the problem could be represented graphically.

Manuscript 3-word problem: *"Mr. Beni buys a 12,000,000 rupiahs motorcycle. He repairs this motorcycle at the cost of 750,000 rupiahs. Mr Beni wants to sell this motorcycle and he wants to get 22.5% profit although being paid in credit per month for a year. How much is the selling price of the motorcycle? How much money should be paid each month by the buyer? [rupiah is the Indonesian currency."*

Fig. 5 Herga Pembelian's response to a written word problem task

Figure 5 above represents the three-step word problem to get the selling price of the motorcycle. The learner, Herga Pembelian's response, shows how they added the buying and repair costs to get a total. On the other side, the percentage increase of 22.5% shows the missing monthly instalment to be calculated. The learner could not arrive at a correct answer, which suggests that the learner had challenges with structuring, representing, and computing the AWP to get an answer. Keesy (2011) comprehends the finding that the effects of using drawing as a problem-solving strategy are unknown for middle school learners in a small rural school system. In this view, there is a need to expose primary school learners to interpret AWP tasks and present representations that can help them make sense of numerical notations. Then, they can solve the numerical problem using procedural knowledge to get the correct answer.
Manuscript 4 also required learners to read the word problem and represent the problem graphically and symbolically. See the word problem below.
"I bought a box of coloured pencils. At home I had pencils, now I have 20 in total. How many pencils are there in total".

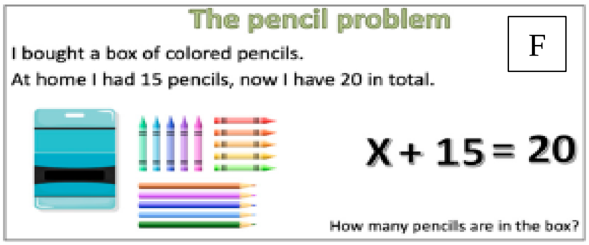

Fig. 6 Extract from Ayala-Altamirano problem session 6 (2022, page 6)

Figure 6 shows the problem as presented during the teaching and learning of numbers. When the problem was presented, learners had different responses, and they were allowed to discuss it in class and come up with a representation of the problem. However, there were two unknowns from the responses before the discussion in class. The symbolic and graphical representation was incorrect since there were two unknowns. However, there is no clear indication of how the respondents identified 15 pencils at home. This suggests that a learner wanted one unknown to get to the number notation. From the different AWP data used in this study, the learners' responses showed that they had difficulties in representing the symbolic, graphical, and language in ways that display their understanding of number notation with the operations used in the problem situation from the different articles. This suggests that learners have difficulties in solving AWP by representing the represented situation in words-to-number notation. Some authors further state that as learners progress in primary school levels, they overcome this challenge.

DISCUSSION

The study investigated how learners solve arithmetic standard word problems involving representations and/or language for number sense in primary school through a literature review. The study found that learners were able to represent AWP tasks with one unknown to represent the situation graphically and symbolically. However, learners struggled to represent the situation symbolically when given two unknown values. The most difficult part of the AWP was finding the correct answer to the given word problem.

The overall display of representations in the reviewed manuscripts showed that solving AWP tasks presented in language was not difficult when they presented situations generally, but they struggled to represent the situation, including mathematical operations used in the situation. Xin et al. (2023) comprehend that selecting and applying appropriate arithmetic operations is important to the success in solving a problem. The use of the word "How far?" the learner added in the AWP; the learner needed to connect the end of the situation by using relational understanding.

CONCLUSION

This study reviewed literature from four manuscripts that focused on solving AWP in primary schools involving representations and language with English second-language speakers. The manuscripts presented the AWP in English, expecting learners to represent them symbolically and graphically. From the manuscripts, a word problem was presented in the Indonesian language, and learners had to respond symbolically and graphically. However, there were no differences identified in the second additional language and first language representation of AWP.

According to the Department of Basic Education (2011), learners solve problems involving whole numbers, including comparing two or more quantities of the same kind, different kinds, and grouping and equal sharing with remainders. Therefore, primary school learners are expected to represent AWP graphically, symbolically, and verbally. However, they still need to be assisted in representing verbal representation to either symbolic or pictorial representation to represent the number notation situation and compare it with symbolic and or pictorial situations. This study concludes that symbolic and pictorial situations should include language to make numbers meaningful to learners in primary schools.

DECLARATION OF CONFLICT

The authors state that they have no financial interests or personal relationships that could have influenced the work presented in this paper.

REFERENCES

1. Anwar, R. B. & Rahmawati, D. (2017). Symbolic and Verbal Representation Process of Student in Solving Mathematics Problem Based Polya's Stages. *International Education Studies;* 10(10), https://doi.org/10.5539/ies.v10n10p20
2. Assapun, S., & Thummaphan, P. (2023). Assessing the effectiveness of board game-based learning for enhancing problem-solving competency of lower secondary students. *International Journal of Instruction*, 16(2), 511-532. https://doi.org/10.29333/iji.2023.16228a
3. Ayala-Altamirano, C., Pinto, E., Molina, M., Cañadas, M. C. (2022). Interacting with intermediate quantities through arithmetic word problems: Tasks to promote algebraic thinking at elementary school, *Mathematics*, 10(1), 2229. https://doi.org/10.3390/math10132229

4. Boud, D., Cohen, R., & Walker, D. (1993). *Introduction: Understanding learning from experience*. In D. Boud, R. Cohen & D. Walker (Eds.), Using experience for learning (pp. 1-17). Bristol, PA: Society for Research into Higher Education.
5. Brannon, E. M. (2005). The independence of language and mathematical reasoning. *PNAS*, 102(9), 3177–3178. https://doi.org/10.1073/pnas.0500328102
6. Capone R, Filiberti F, Lemmo A. (2021). Analyzing Difficulties in Arithmetic Word Problem Solving: An Epistemological Case Study in Primary School. *Education Sciences*. 11(10), 1-20. https://doi.org/10.3390/educsci11100596
7. Castleberry, A., & Nolen, A. (2018). Thematic Analysis of Qualitative Research Data: Is It as Easy as It Sounds? *Currents in Pharmacy Teaching and Learning*, 10, 807-815. https://doi.org/10.1016/j.cptl.2018.03.019
8. Creswell, J. W., & Creswell, J. D. (2018) Research design: qualitative, quantitative and mixed methods approaches. 5th Edition. California: SAGE
9. Darockzy, G., Wolska, M., Meurers, W. D., & Nuerk, H. C. (2015). Word problems: A review of linguistic and numerical factors contributing to their difficulty. *Sec. Developmental Psychology*, 6, 1-13. https://doi.org/10.3389/fpsyg.2015.00348
10. Davis, E. (2023). Mathematics, word problems, common sense, and artificial intelligence. *arXiv preprint arXiv:2301.09723*.
11. Department of Basic Education (2011). *Curriculum and assessments policy statements, Grades 4-6 mathematics.* Department of Basic Education.
12. Dewi, L., Susilana, R., Setiawan, B., Alias, N., & Zulnaidi, H., (2023). A proposed problem centered thinking skill (PCTS) model at secondary schools in Indonesia and Malaysia. International *Journal of Instruction*, 16(3), 615-638. https://doi.org/10.29333/iji.2023.16333a
13. Elia, I. (2020). Word problem solving and pictorial representations: insights from an exploratory study in kindergarten. *ZDM* 52:17–31 https://doi.org/10.1007/s11858-019-01113-0
14. Fatmanissa, N & Kusnandi (2017). The linguistic challenges of Mathematics word problems: A research and literature review. *Malaysian Journal of Learning and Instruction (MJLI), Special issue on Graduate Students Research on Education*, 73-92.
15. Graven, M., Venkat, H., Westaway, L. & Tshesane, H. (2013). Place value without number sense: Exploring the need for mental mathematical skills assessment within the Annual National Assessments. *South African Journal of Childhood Education* 3(2), 131-143.
16. Hayashi, Y., Tsudaka, N., Iwai, K., & Hirashima, T. (2021). Problem posing with representation conversion model for learning the condition of addition and subtraction word problems. *Proceedings of the 29th International Conference on Computers in Education. Asia-Pacific Society for Computers in Education*.
17. Hickendorff, M (2021). The demands of simple and complex arithmetic word problems on language and cognitive resources. *Frontiers in Psychology*. 1-12. https://doi.org/10.3389/fpsyg.2021.727761
18. Jan, S, & Rodriguez, S. (2012). students' difficulties in comprehending mathematical word problems in English language learning contexts. *International researchers*. 1(3), 152-159.
19. Keesy, M. A. (2011). *Word problems: the effects of learner generated drawings on problem solving.* Unpublished Doctor of Philosophy. Capella University.
20. Knight, L., Goldsworthy, K., Visser, S., Hall, C. (2023). Thinking beyond circles: Developing visual research methods for circularity in design education. PLATE Conference Espoo, Finland, 31 May - 2 June 2023
21. Kuhn, J. T. & Holling, H. (2014). Number sense or working memory? The effect of two computer-based trainings on mathematical skills in elementary school. *Advances in cognitive technology*, 10(2), 59-67.
22. Kwok, M., Welder, R. M., Moore J., & Williams, A. M. (2022). Applying systematic functional linguistics to unpack the language of additive word problems. *International Journal of Science and Mathematics Education*, 20(Suppl 1), 163–186. https://doi.org/10.1007/s10763-022-10290-6
23. Mandal, S., & Naskar, S. K. (2019). Solving Arithmetic Mathematical Word Problems: A Review and Recent Advancements. *In Advances in Intelligent Systems and Computing*, 699), https://doi.org/10.1007/978-981-10-7590-2_7
24. Mandal, S., Acharya, S., & Basak, R. (2022). Solving arithmetic word problems using natural language processing and rule-based classification. *International journal of intelligent systems and applications in engineering*, 10(1), 87-97.
25. Nasrun, Prahmana, R. C. I., & Akib, I. (2023). The Students' Representative Processes in Solving Mathematical Word Problems. *Knowledge*, *3*(1), 70–79. http://dx.doi.org/10.3390/knowledge3010006
26. National Research Council, (2000). *How people learn: brain, mind, experience, and school: expanded edition*. National Academic Press.
27. Njagi, M. W. (2015). Language issues on mathematics achievement. *International Journal of Education and Research*, 3(6), 167-178.
28. North, K., Zampieri, M., & Shardlow, M. (2023). Lexical complexity prediction: An overview. *ACM comput surv*, 55(9), 1-42. https://doi.org/10.1145/3557885

29. Pallotti, G. (2014). A simple view of linguistic complexity. *Second Language Research,* 31(1), 117–134.
30. Payne, J., & Rathmell, E. C. (1975*). Mathematics learning in early childhood: Number and numeration.* National Council of Teachers of Mathematics Yearbook (Vol. 37, pp. 125–160).
31. Planner, R. J. & Sterelny, K. (2021). *From signal to symbol: The evolution of language*. The Massachusetts Institute of Technology.
32. Polotskaia, E. & Savard, A. (2020). Some multiplicative structures in elementary education: a view from relational paradigm. *Educational studies in mathematics*, 106, 447-469. https://doi.org/10.1007/s10649-020-09979-8
33. Rahmah, F., Subanji, & Irawati, S. (2019). *Mathematical representation analysis of students in solving mathematics problems.* 5th International Symposium on Mathematics Education and Innovation (ISMEI) 13–14 November 2018, Yogyakarta, Indonesia
34. Rochat & Callaghan (2005). *What drives symbolic development? The case of pictorial comprehension and production*. In L. L. Namy (Ed.), Symbol use and symbolic representation: Developmental and comparative perspectives. Lawrence Erlbaum Associates
35. Rossiter, M. P. (2023). 'What do you mean versus what you say' – exploring the role of language and culture in European students' interpretation on feedback. *Assessment and Evaluation in Higher Education*, 48(4), 544-555. https://doi.org/10.1080/02602938.2022.2097197
36. Sanwidi, A. (2018). Students' representation in solving word problems. *Journal of Mathematics Education*, 7(2), 147-154. DOI 10.22460/infinity.v7i2.p147-154
37. Roy, S., & Roth, D. (2016). Solving general arithmetic word problems. *arXiv preprint arXiv:1608.01413*.
38. Schumacher, R. F. & Fuchs, L. S., (2012, p. 608). Does understanding relational terminology mediate effects of intervention on compare word problems? *Journal of Experimental Child Psychology*, 111(4), 607-628. https://doi.org/10.1016/j.jecp.2011.12.001
39. Sousa, D.A. 2015. *How the Brain Learns Mathematics*. (2nd ed.). Thousand Oaks: Corwin Press.
40. Strohmaier, A.R., Reinhold, F., Hofer, S., Berkowitz, M., Vogel-Heuser, B., & Reiss, K. (2021). Different complex word problems require different combinations of cognitive skills. *Educ Stud Math,* 109, 89–114. https://doi.org/10.1007/s10649-021-10079-4
41. Tshesane, H., & Venkat, H. (2023). Coding Additive Word Problem-solving to see Shifts Around an Intervention, African Journal of Research in Mathematics, Science and Technology Education, DOI: 10.1080/18117295.2023.2226548
42. Verschaffel, L., Schukajlow, S., Star, J., & Van Dooren, W. (2020). Word problems in mathematics education: A survey, *ZDM,* 52, 1-16. https://doi.org/10.1007/s11858-020-01130-4
43. Wang, S., Wei, Z., Xu, J., & Fan, Z. (2023). *Unifying Structure Reasoning and Language Model Pre-training for Complex Reasoning*. Cornell University. https://doi.org/10.48550/arXiv.2301.08913
44. Wrenn, J. & Wrenn, B. 2009. Enhancing learning by integrating theory and practice. International Journal of Teaching and Learning in Higher Education, 21(2):258-265.
45. Xin, Y. P., Kim, S. J., Lei, Q., Liu, B. Y., Wei, S., Kastberg, S. E., & Chen, Y. V. (2023). The Effect of Model-Based Problem Solving on the Performance of Students Who are Struggling in Mathematics. *The Journal of Special Education*, 1-12. 00224669231157032.
46. Yin R, K. (2016). *Qualitative research from start to finish*. 2nd Edition. New York: The Guilford Press.
47. Ziadat, A. H. (2022). Sketchnote and working memory to improve mathematical word problem solving among children with dyscalculia. *International Journal of Instruction*, *15*(1), 509- 526.