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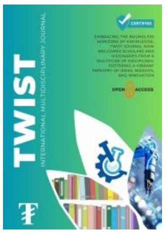

Analysis of Univariate Stable Distributions using Fractional Calculus and Real-World Applications

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Abstract
In this study we see, how are apply fractional calculus to demonstrate a relationship between the stable distributions? To do so, it is necessary to first formulate the appropriate fractional diffusion equations. Standard diffusion equations can be extended into what are called fractional diffusion equations. This enlargement can be accomplished by considering either a time or a space derivative on a fractional scale. The fractional derivative is used to extend the reach of standard diffusion equations in this article. Obtain some analytic-numerical approximations for the PDF of the univariate stable distributions by employing some analytic-numerical approaches, such as the homotopy perturbation method, the Adomian decomposition method, and the variational iteration method, which are employed to solve partial differential equations (PDEs) and perform stability analysis. By using fractional calculus, one can precisely manage a wide variety of mathematical models. It is applied to a wide variety of problems, including those involving turbulence, pollution, population growth and spread, landscape development, medical imaging, and complex systems.

Keywords
Fractional Derivative, Stable Distributions, Univariate, Stability, Diffusion Equations

INTRODUCTION

In this paper looks, how fractional calculus can be used in economics, finance, and modeling. We start with a short introduction to the field of fractional calculus, which covers two key points right away: first, the fractional paradigm is important in science, especially in the fields of finance and economics; and second, the fractional paradigm is important for calculus and random processes.

Calculus with fractional denominators extends the capabilities of classical calculus. The ability to write differential equations that relate variables and their rates of change has made calculus a crucial tool for modern science. Equations with a difference started the contemporary era of quantitative theory in science. Differential equations are especially useful because they simplify the description of physical phenomena at the local level, such as how heat travels along a rod after being applied to one end. Locally, the heat equation is relatively simple: at any point on the rod, the time derivative of the temperature is proportional to the second space derivative, even if the actual description of the propagation of temperature can be quite complex depending on the source of heat.

This is why differential equations work so well in science: things simplify themselves locally, and it's possible to come up with a general law that is easy to understand and very effective.

Yet, not all problems are confined to a single location. There are a lot of challenges of a global nature in the fields of physics, engineering, and economics. Take, for instance, variational problems, the aim of which is to maximize the value of a function—that is, to locate the point at which the function performs at its best in a domain consisting of functions. The translation of variational issues into integral or differential equations has been a significant achievement in mathematics. This is despite the fact that variational problems do not involve local variables. In spite of this, the formalism of fractional calculus is a very helpful instrument for the resolution of issues involving variation.

For instance, Abel's integral equation solves the tautochrone problem. Fractional derivatives can solve Abel integral equations. Several articles written in the last few years have shown that there is a strong link between the calculus of variations and the fractional calculus (seeGorenflo et al. [7] and Mainardi et al. [14]).

The calculus of variations is one of the most important ways to use math in science and engineering. For example, classical dynamics can be described in terms of the task of maximizing Hamiltonian functionals. This can be done by using the term "variational." Variational ideas provide the foundation of control theory. Formulating and resolving issues involving variation need the application of fractional calculus. Fractional calculus is intriguing because it has a non-local feature (in science and engineering).

In financial risk management, probability distribution functions with "fat tails" are common, and fractional calculus can be used to figure them out. We will see that the solutions of certain fractional partial differential equations have stable, fat-tailed distributions. Unfortunately, there are no closed formulas that can adequately explain these distributions. There must be approximations made at the numerical level. Fat-tailed stable distributions can be computed numerically with the help of fractional calculus.

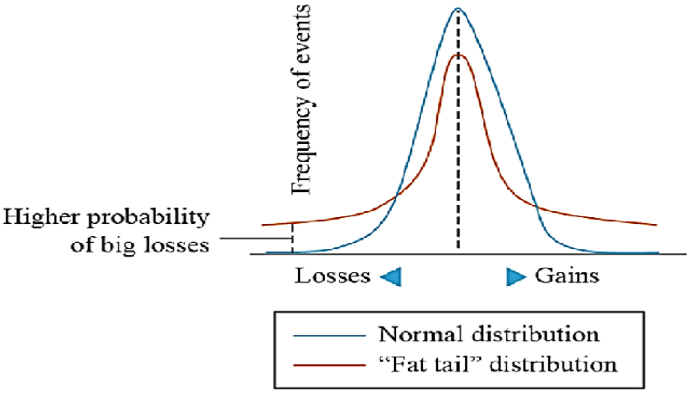

Fig. 1 Fat-tail

Pricing financial contracts like options has been shown to benefit a lot from the solution of fractional partial differential equations. But the traditional Black-Scholes equations (see Black and Scholes [1]) don't work well in many real-world situations.

Stochastic processes, which are fractional like mathematics, are used in banking and economics to simulate volatility, interest rates, and high-frequency data.

Several papers on heavy tail distributions during the 1960s, and there have been many more since then. These studies support the idea that the heavy tail is a characteristic about financial time series. When it comes to empirical samples, the variance doesn't change based on how big the sample is, but the variance for stable distributions is always the same. Diffusion equations are extended to fractional order in one application of fractional calculus to establish a relationship between stable and tempered stable distributions.

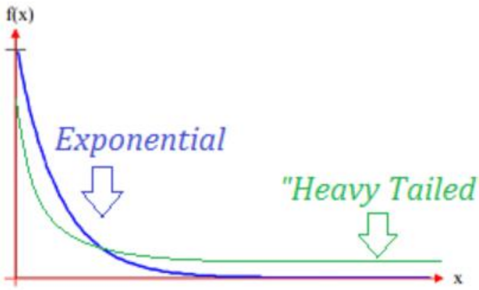

Fig. 2 Heavy Tailed

We cover the ideas and properties of the most well-known definitions of fractional calculus as well as the definitions that will be utilized in this study because there are different ways to extend ordinary calculus to fractional calculus.

Some important definitions for fractional calculus are the Riemann-Liouville fractional derivative, Caputo fractional derivative, Grünwald-Letnikov fractional derivative, and fractional derivative based on the Fourier transform are discussed (see Samko et al. [19], Kilbas et al. [11], Podlubny [17], Hashemiparast and Fallahgoul [8-9],Fallahgoul et al. [5-6]).

STABLE DISTRIBUTION

We See in this section to use of fractional calculus to build a relationship with the stable distributions. The fundamental solution of the appropriate fractional diffusion equations, which are defined to achieve this, provides the PDF for the univariate stable distributions. Extensions of standard diffusion equations are fractional diffusion equations. Consideration of a fractional derivative in either time or space can be used to carry out this extension. In this study,apply the fractional derivative in space to extend standard diffusion equations. For the second part, obtain some analytic-numerical approximations for the PDF of the univariate stable distributions by employing some analytic-numerical approaches, such as the homotopy perturbation method, the Adomian decomposition method, and the variational iteration method, which are used for solving partial differential equations (PDEs) and analyzing of stability.

Now, next is demonstrate the relationship between the univariate stable distribution and fractional calculus. The PDF of the univariate stable distributions is found by various authors, whose fundamental solutions are described in further detail (see Fallahgoul et al. [4-5]).

We consider the following equation, illustrated by Fallahgoul et al. [4-5].

$$\frac{\partial u}{\partial t} = -\frac{1+\beta}{2c}\frac{\partial^{\alpha}}{\partial x^{\alpha}}u(x,t) - \frac{1-\beta}{2c}\frac{\partial^{\alpha}}{\partial(-x)^{\alpha}}u(x,t) + \mu\frac{\partial}{\partial x}u(x,t), \qquad (1)$$

equation (1) is called a fractional diffusion PDE, where $0 < \alpha \leq 2, \alpha \neq 1, -1 \leq \beta \leq 1, \ -\infty < \mu < \infty, \ c = \cos\frac{\alpha\pi}{2}$ and $s = \sin\frac{\alpha\pi}{2}$. Assume to be$H(\omega,t)$ be the Fourier transform of $u(x,t)$ with respect to t, can write an equation (1) in form of an initial value problem

$$\frac{\partial H}{\partial t} = -\frac{1+\beta}{2c}(i\omega)^{\alpha}H - \frac{1-\beta}{2c}(-i\omega)^{\alpha}H + (i\mu\omega)H. \qquad (2)$$

Assume$u(x,0) = \delta(x)$, is an initial value, then $H(\omega,0) = 1$. So the solution of an equation (2) can be expressed as

$$H(\omega,t) = \exp\left\{-\frac{1+\beta}{2c}(i\omega)^{\alpha}t - \frac{1-\beta}{2c}(-i\omega)^{\alpha}t + (i\mu\omega)\right\}. \qquad (3)$$

Equation (3) of the fractional PDE is equivalent to equation (1)

$$\frac{\partial u}{\partial t} = -\frac{\beta}{c}\frac{\partial^{\alpha}}{\partial x^{\alpha}} + (1-\beta)\frac{\partial^{\alpha}}{\partial(-x)^{\alpha}}u(x,t) + \mu\frac{\partial}{\partial x}u(x,t), \qquad (4)$$

where $(x,0) = u_0(x), -\infty < x < \infty$, $t > 0$. So we can find the Fourier transform from the fundamental solution of (4), written as

$$H(\omega,t) = \exp\left\{-|\omega|^{\alpha}t - i\beta\operatorname{sign}(\omega)\tan\frac{\alpha\pi}{2}|\omega|^{\alpha}t + i\mu\omega t\right\}. \qquad (5)$$

At $\alpha \neq 1$, the univariate stable distribution's characteristic function (CF) is identical to an equation (5). The resulting PDF for the univariate normal distribution is u(x,t) (i.e., $S_{\alpha}\left(t^{\frac{1}{\alpha}}, \beta, \mu t\right)$)(see [4-5]).

Finally, obtain different analytic-numerical approximations for the PDF of the univariate stable distribution using techniques like the homotopy perturbation method (HPM), the Adomian decomposition method (ADM), and the variational iteration method (VIM), (see Fallahgoul et al. [4-5], He [10]). Using a finite difference method or a finite element method can improve the approximations' precision. The issue persists and merits more research.

Homotopy Perturbation Method

Using the homotopy perturbation method, the solution of an equation (1) is given by

$$v = v_0 + v_1p^1 + v_2p^2 + v_3p^3 + \cdots,$$

where $v_0 = u(x,0) = \delta(x)$. By applying the Homotopy perturbation method, one can show that

$$v_1(x,t) = \left(\frac{d_1 + (-1)^{\alpha}d_2}{2\Gamma(-\alpha)}\right)x^{-\alpha-1} \times t$$

$$v_2(x,t) = \left(\left(\frac{d_1^2 + (-1)^{\alpha}d_1d_2 + d_2^2}{2\Gamma(-2\alpha)}\right)x^{-2\alpha-1} + \left(\frac{d_1 + (-1)^{\alpha}d_2}{2\Gamma(-\alpha-1)}\right)x^{-\alpha-2}\right)\frac{1}{2}t^2.$$

We can derive the following recurrent relation

$$v_j = \int_0^t \left(d_1\frac{\partial^{\alpha}v_{n-1}}{\partial x^{\alpha}} + d_2\frac{\partial^{\alpha}v_{n-1}}{\partial(-x)^{\alpha}} + \mu\frac{\partial v_{n-1}}{\partial x}\right)dt$$

using the finite element or finite difference an approach. This is a persistent issue that merits more research, where $j = 3,4,5,\cdots$

$$u_0(x,t) = v_0(x) = \delta(x)$$

$$u_1(x,t) = v_0 + v_1 = \delta(x) + \left(\frac{d_1 + (-1)^{\alpha}d_2}{2\Gamma(-\alpha)}\right)x^{-\alpha-1} \times t$$

$$u_2(x,t) = v_0 + v_1 + v_2 = \delta(x) + \left(\frac{d_1 + (-1)^{\alpha}d_2}{2\Gamma(-\alpha)}\right)x^{-\alpha-1} \times t$$

$$+ \left(\left(\frac{d_1^2 + (-1)^{\alpha}d_1d_2 + d_2^2}{2\Gamma(-2\alpha)}\right)x^{-2\alpha-1} + \left(\frac{d_1 + (-1)^{\alpha}d_2}{2\Gamma(-\alpha-1)}\right)x^{-\alpha-2}\right)\frac{1}{2}t^2$$

This is how we can locate the remaining portion. We assume $u(x,t) = \lim_{n\to\infty}u_n(x,t)$.

This way we can calculate more terms, and we prove that $u(x,t)$ is the PDF for the univariate stable distribution with respect to x; the PDF for the univariate stable distribution is,

$$p(x) = u(x,t) = \lim_{n\to\infty}u_n(x,t) = S_{\alpha}\left(t^{\frac{1}{\alpha}}, \beta, \mu t\right).$$

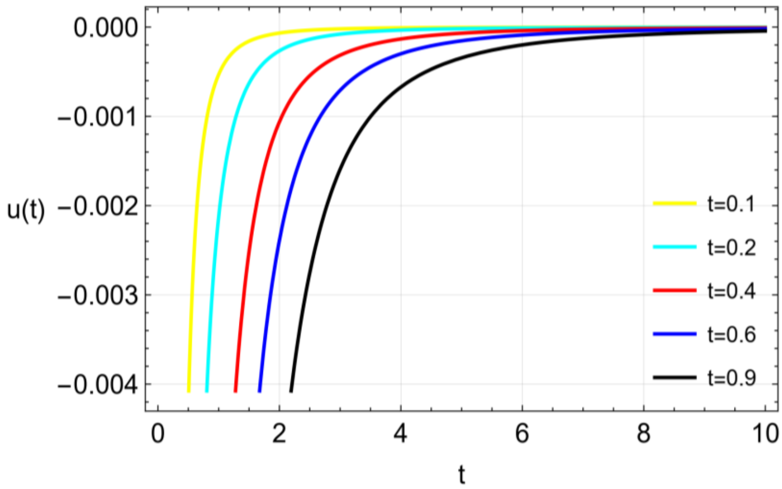

Fig. 1 Stability (a)

We observe in Figure 1. Once we fixed the value of t to be one of the following: 0.1, 0.2, 0.4, 0.6, or 0.9, we saw that the function $u(t)$ would exhibit a stable situation after some value on 0.000 beginning with -0.004 or higher negative value. This indicates that the function $u(t)$ is demonstrating stability.

Adomian Decomposition Method

By the Adomian Decomposition Method, we can find the solution of an equation (1) in recurrence form

$$u_0 = u(x,0) = \delta(x)$$

$$u_{k+1} = \int_0^t \left(D_1 \frac{\partial^\alpha u_k}{\partial x^\alpha} + D_2 \frac{\partial^\alpha u_k}{\partial(-x)^\alpha} + \mu \frac{\partial u_k}{\partial x} u_k \right) dt, \quad k = 1,2,\cdots$$

where $D_1 = -\frac{1+\beta}{2c}$ and $D_2 = -\frac{1-\beta}{2c}$.

Hence, this calculationsarrive at the recurring relationship shown below.

$$u_j = \int_0^t \left(D_1 \frac{\partial^\alpha u_{j-1}}{\partial x^\alpha} + D_2 \frac{\partial^\alpha u_{j-1}}{\partial(-x)^\alpha} + \mu \frac{\partial u_{j-1}}{\partial x} \right) dt$$

for $j = 3,4,5,\cdots$. As a result, the analytic-numerical approximation for the PDF of the univariate stable distribution is$p(x) = u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$.

Variational Iteration Method

We will see the solution of an equation (1) by Variational Iteration Method,

$$u_{n+1} = u_n + \int_0^t \lambda \left(\frac{\partial u_n}{\partial s} - D_1 \frac{\partial^\alpha v_n}{\partial x^\alpha} - D_2 \frac{\partial^\alpha v_n}{\partial(-x)^\alpha} - \mu \frac{\partial v_n}{\partial x} \right) ds$$

choose $u_0(x,t) = u(x,0) = \delta(x)$. We get$u_1$ and u_2

$$u_1(x,t) = \left(\frac{D_1 + (-1)^\alpha D_2}{2\Gamma(-\alpha)} \right) x^{-\alpha-1} \times t.$$

and

$$u_2(x,t) = \left(\frac{D_1^2 + (-1)^\alpha D_1 D_2 + D_2^2}{2\Gamma(-2\alpha)} \right) x^{-2\alpha-1} + + \left(\frac{D_1 + (-1)^\alpha D_2}{2\Gamma(-\alpha-1)} \right) x^{-\alpha-2} \Big) \frac{1}{2} t^2$$

respectively.

Hence, we arrive the recurrent relationship shown below.

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \left(\frac{\partial u_n}{\partial s} - D_1 \frac{\partial^\alpha u_n}{\partial x^\alpha} - D_2 \frac{\partial^\alpha u_n}{\partial(-x)^\alpha} - \mu \frac{\partial u_n}{\partial x} \right) ds$$

The remaining Variational Iteration Method components can be acquired in this way. If compute further terms and demonstrate that $u(x,t) = \lim_{n\to\infty} u_n(x,t)$ is the PDF for the univariate stable distribution with respect to x. (i.e., the solution converges to the PDF of the univariate stable distribution).

We observe in Figure 2 after analyzing, once we fixed the value of t to be one of the following: 0.1, 0.2, 0.4, 0.6, or 0.9, we saw that the function $u(t)$ would exhibit a stable situation after some value on 0.000 beginning with -0.05 or higher negative value. This indicates that the function $u(t)$ is demonstrating stability.

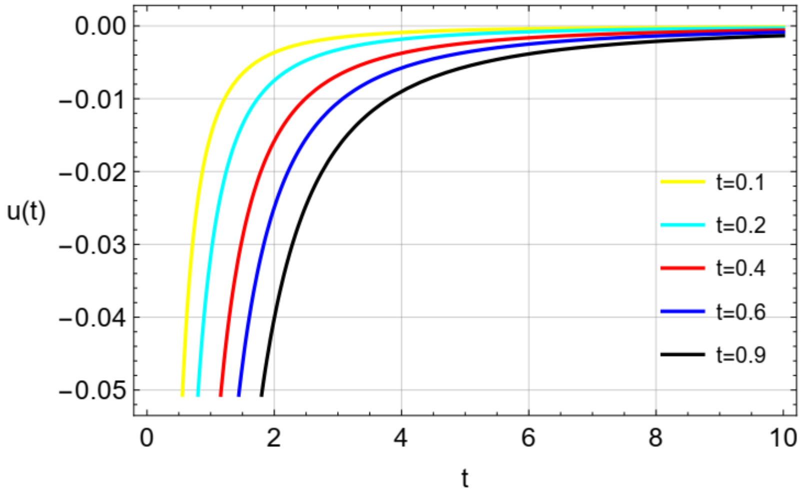

Fig. 2 Stability (b)

THE APPLICATION OF AN ISSUE IN THE REAL WORLD

A wide range of mathematical models can be accurately governed by fractional order differential equations. Fractional order differential equations provide an accurate governing mechanism for a wide variety of mathematical models. Liouville, Riemann, Leibniz, and others are credited with carrying out some of the earliest systematic research [15, 21]. Fractional calculus has, for a significant portion of history, been viewed as a domain of pure mathematics devoid of any practical applications. Yet, this state of affairs has been significantly altered during the past few decades. It has been discovered that fractional calculus can be helpful and even powerful, and an overview of the simple history of fractional calculus, especially in relation to applications, can be found in Machado et al. [13]. Machado et al. [13] also contain some examples of how fractional calculus has been used. The field of fractional calculus and its applications is currently undergoing a period of rapid progress, with applications in the real world becoming increasingly convincing [12, 16]. The study of fractional differentiation and integration is inherently multidisciplinary, and its applications can be found in a wide variety of fields and contexts. These fields include continuum mechanics, elasticity, signal analysis, quantum mechanics, bioengineering, biomedicine, financial systems, social systems, pollution control, turbulence, population growth and dispersal, landscape evolution, medical imaging, and complex systems, as well as some other branches of pure and applied mathematics [22, 23, 24, 25, 26, 27].

One of the problems among these, whichwe will have discussed here, is that stochastic processes have been suggested as possible replacements for geometric Brownian motion as a way to price options. The exponential L'evy process, which contributes to a diverse class of stochastic processes, does a good job of describing the stylized facts that have been found in relation to the distribution of asset prices.

Important examples of the exponential L'evy process include the variance-gamma process, the normal inverse Gaussian process, the stable process, the log-stable process, and the CGMY process. These processes are jump processes in their purest form and have an endless amount of activity. It is important to point out that the Brownian motion is an illustration of the L'evy process, [see, Cont and Tankov [3]]. In this section, we will talk about how the CGMY process can be applied to the pricing of options.

Definition (Carr et al. [2]). Let (X_t) t ≥ 0, be a stochastic process in probability space. X_t is theL´evy process if it has the $X_0 = 0$ andX_t has independent increments, stationary increments, and is stochastically continuous. In more details, $\lim_{s\to t}$ P $[|X_s - X_t| > a] = 0$∀t ≥ 0 and a > 0. $X_t$is a cadlag.

In practice, the L'evy approach relies heavily on the characteristic function. Using the L'evy-Khintchine formula (see Cont and Tankov [3]), we can get the characteristic function of the univariate L'evy process X_t as given

$F(u;\, X_t) = e^{f(u,X_t)t}$.

$$\text{Where,} f(u, X_t) = \left(i\gamma u - \frac{1}{2}\sigma^2 u^2 + \int_{-\infty}^{+\infty} (e^{iux} - 1 - iux1_{|x|\leq 1})\vartheta(dx)\right). \qquad (6)$$

Where $f(u, X_t)$ is the characteristic exponent and $1_{|x|}$is an indicator function. If consider $\vartheta(dx) = 0$, then equation (6) is equal to the characteristic function of Brownian motion. And if we consider $\sigma = 0$ in equation (6), L'evy process is equal to the pure-jump process.

Second, we will look CGMY Process, which has the following equation is as below (see, Samorodnitsky and Taqqu [20] and Rachev et al. [18]),

$$\vartheta(dx) = \left(C\left(e^{-Mx}/_{x^{1+Y}}\right)1_{x>0} + C\left(e^{-G|x|}/_{|x|^{1+Y}}\right)1_{x<0}\right). \qquad (7)$$

Where, $C, G, M > 0, 0 \leq Y < 2.$ These factors are the characteristics of the CGMY process. G and M, respectively, display the left tail's decay rate and the right tail's decay rate. The exponential exponent Yhas a significant impact on how quickly the tail decay or the fine structure of the process.

In order to obtain the CGMY processes, one must first temper the tails of the stable processes. Because of this, the process is sometimes referred to as the traditional tempered stable method. The L'evy metric for the stable process has been shown to have some sort of connection to the CGMY process. One can achieve the L'evy measure of the stable process by first ensuring that $G = M = 0$ and then selecting distinct values for parameter C for the left and right sides of the system. The fact that there is just the initial moment that exists is a limitation of the stable process. A finite moment of any order can be generated by first obtaining a stable process and then modifying its tails. In place of the CGMY method, some researchers opt to make use of the tempered stable method.

We are consider equation (7), and assume $1_{x<0} = 1_{x>0} = 1$, $G = M = 0$,

Case 1: when we consider $|x|$ = +ive value only, so equation (7) will be become

$$\vartheta(dx) = \left(\left({}^{2C}/_{x^{1+Y}}\right)\right) \tag{8}$$

Case 2: If we take $|x|$ = negative value then we find $\vartheta(dx) = 0$.

We calculate $\vartheta(dx)$ by equation (8) on different parameter value, and analyzing graphical to see stability.

Table 1

x	c=150, y=0.3	c=200, y=0.3	c=250, y=0.3	c=300, y=0.3
10	15.03561701	20.04748935	25.05936168	30.07123402
11	13.28344629	17.71126172	22.13907715	26.56689258
12	11.86275702	15.81700935	19.77126169	23.72551403
13	10.69042317	14.25389756	17.81737195	21.38084634
14	9.708559763	12.94474635	16.18093294	19.41711953
15	8.875700069	11.83426676	14.79283345	17.75140014
16	8.161411531	10.88188204	13.60235255	16.32282306
17	7.542887961	10.05718395	12.57147993	15.08577592
18	7.002723812	9.336965083	11.67120635	14.00544762
19	6.527420235	8.703226981	10.87903373	13.05484047
20	6.106357973	8.141810631	10.17726329	12.21271595
21	5.731076051	7.641434735	9.551793419	11.4621521
22	5.39475554	7.193007386	8.991259233	10.78951108
23	5.091843745	6.789124994	8.486406242	10.18368749
24	4.817776407	6.423701876	8.029627344	9.635552813
25	4.568769453	6.091692604	7.614615755	9.137538906
26	4.341660919	5.788881226	7.236101532	8.683321839
27	4.133789533	5.511719378	6.889649222	8.267579067
28	3.942900466	5.257200622	6.571500777	7.885800933
29	3.767071449	5.022761932	6.278452415	7.534142898
30	3.604654325	4.806205767	6.007757208	7.20930865

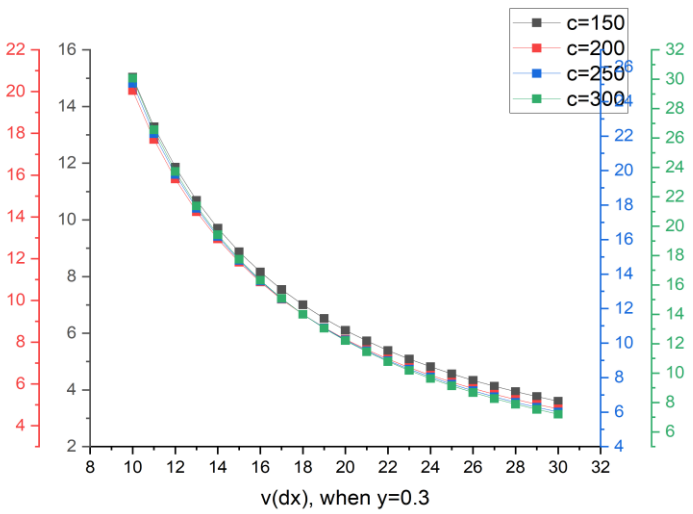

Fig. 1(a)

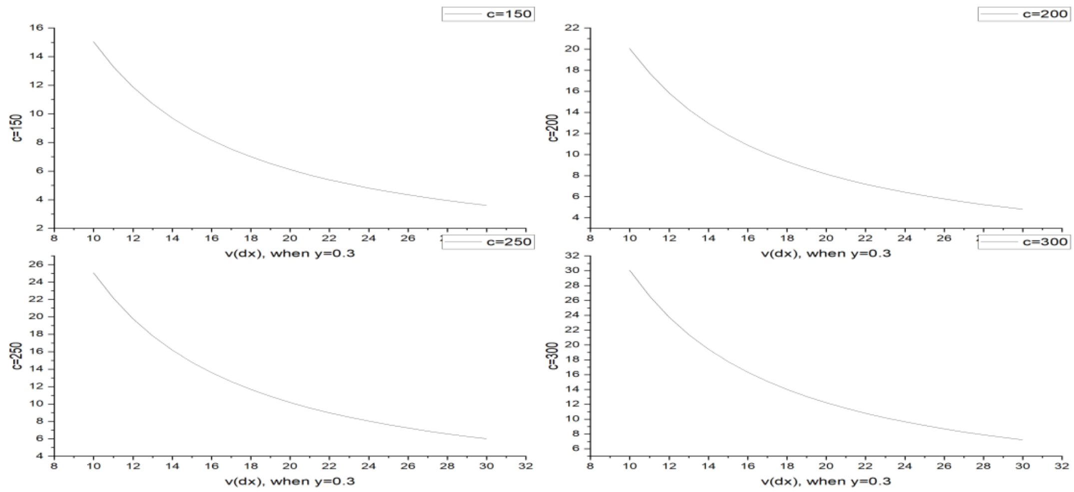

Fig. 1(b)

In Table 1, we consider x =10 to 30 percent, parameter c = 150 to 300, and the value of y = 0.3 years. In Figures 1(a) and 1(b), we can see the stability and fluctuation of$\vartheta(dx)$. So we looked,$\vartheta(dx)$ is an identical decrease.And it is showing positive half tail.

Table 2

x	c=150, y=0.4	c=200, y=0.4	c=250, y=0.4	c=300, y=0.4
10	11.94322	15.92429	19.90536	23.88643
11	10.45133	13.9351	17.41888	20.90266
12	9.252679	12.33691	15.42113	18.50536
13	8.27181	11.02908	13.78635	16.54362
14	7.45662	9.94216	12.4277	14.91324
15	6.770075	9.026767	11.28346	13.54015
16	6.185193	8.246924	10.30866	12.37039
17	5.681889	7.575852	9.469815	11.36378
18	5.244931	6.993241	8.741551	10.48986
19	4.862574	6.483432	8.10429	9.725148
20	4.525632	6.034176	7.54272	9.051265
21	4.226825	5.635766	7.044708	8.45365
22	3.960313	5.280417	6.600522	7.920626
23	3.721365	4.96182	6.202275	7.44273
24	3.50611	4.674813	5.843516	7.01222
25	3.311351	4.415135	5.518919	6.622702
26	3.13443	4.17924	5.22405	6.26886
27	2.973117	3.964156	4.955195	5.946234
28	2.825531	3.767374	4.709218	5.651061
29	2.690073	3.586764	4.483455	5.380146
30	2.565379	3.420505	4.275631	5.130758

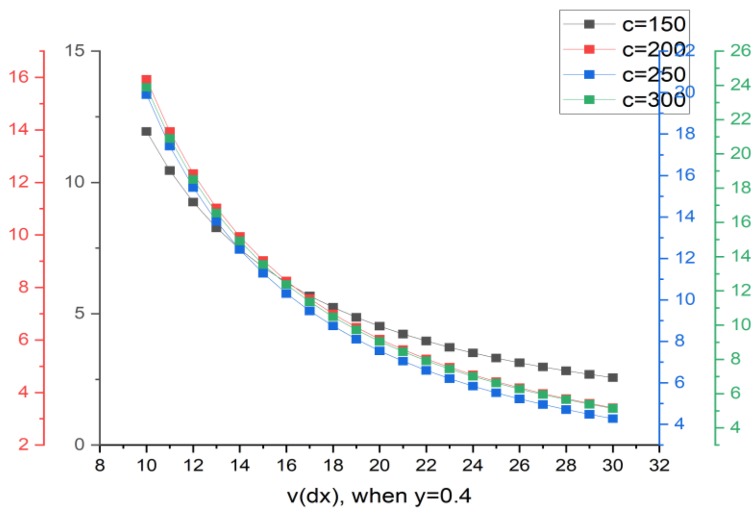

Fig. 2(a)

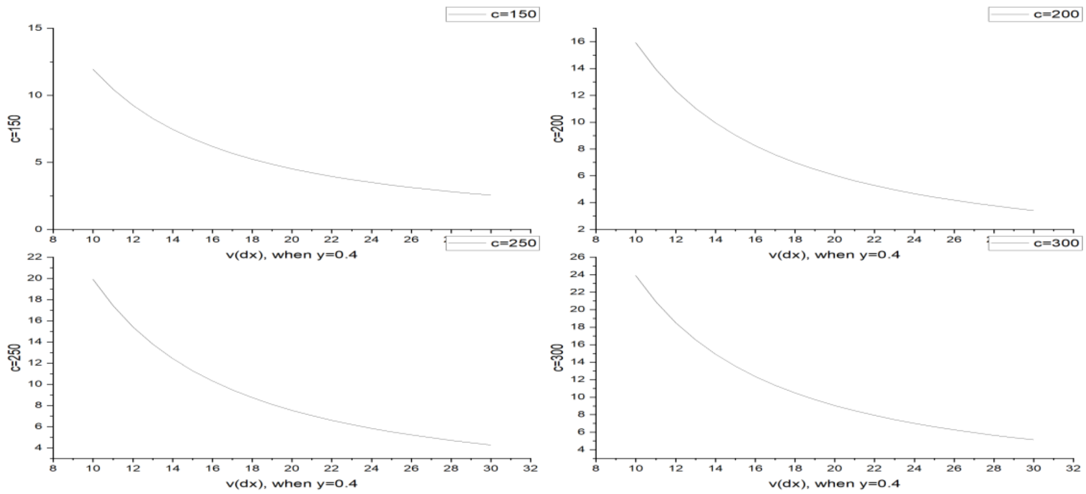

Fig. 2(b)

In Table 2, we consider x =10 to 30 percent, parameter c = 150 to 300, and the value of y = 0.4 years. In Figures 2(a) and 2(b), we can see the stability and fluctuation of $\vartheta(dx)$. So we looked,$\vartheta(dx)$is a greater decrease on the parameters c = 200 and c = 250, and a lesser decrease in the parameter c =150 value. The value of parameter c = 300 is a medium decrease. And it is showing positive half tail.

Table 3

x	c=150, y=0.5	c=200, y=0.5	c=250, y=0.5	c=300, y=0.5
10	15.03561701	20.04748935	25.05936168	30.07123402
11	13.28344629	17.71126172	22.13907715	26.56689258
12	11.86275702	15.81700935	19.77126169	23.72551403
13	10.69042317	14.25389756	17.81737195	21.38084634
14	9.708559763	12.94474635	16.18093294	19.41711953
15	8.875700069	11.83426676	14.79283345	17.75140014
16	8.161411531	10.88188204	13.60235255	16.32282306
17	7.542887961	10.05718395	12.57147993	15.08577592
18	7.002723812	9.336965083	11.67120635	14.00544762
19	6.527420235	8.703226981	10.87903373	13.05484047
20	6.106357973	8.141810631	10.17726329	12.21271595
21	5.731076051	7.641434735	9.551793419	11.4621521
22	5.39475554	7.193007386	8.991259233	10.78951108
23	5.091843745	6.789124994	8.486406242	10.18368749
24	4.817776407	6.423701876	8.029627344	9.635552813
25	4.568769453	6.091692604	7.614615755	9.137538906
26	4.341660919	5.788881226	7.236101532	8.683321839
27	4.133789533	5.511719378	6.889649222	8.267579067
28	3.942900466	5.257200622	6.571500777	7.885800933
29	3.767071449	5.022761932	6.278452415	7.534142898
30	3.604654325	4.806205767	6.007757208	7.20930865

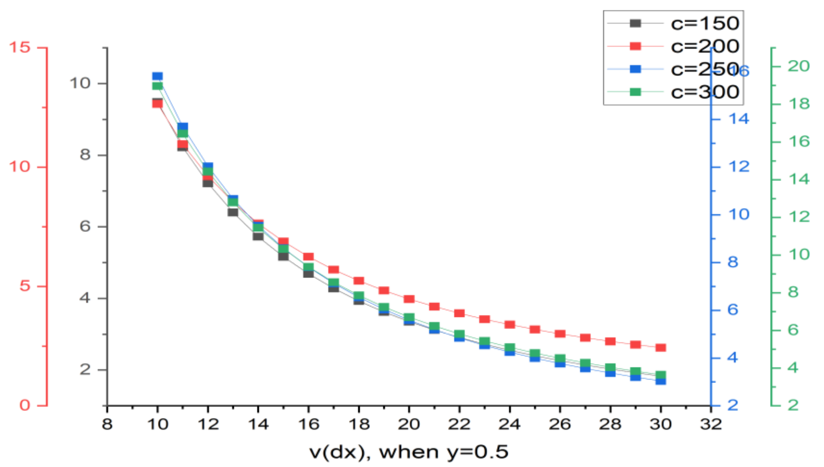

Fig. 3(a)

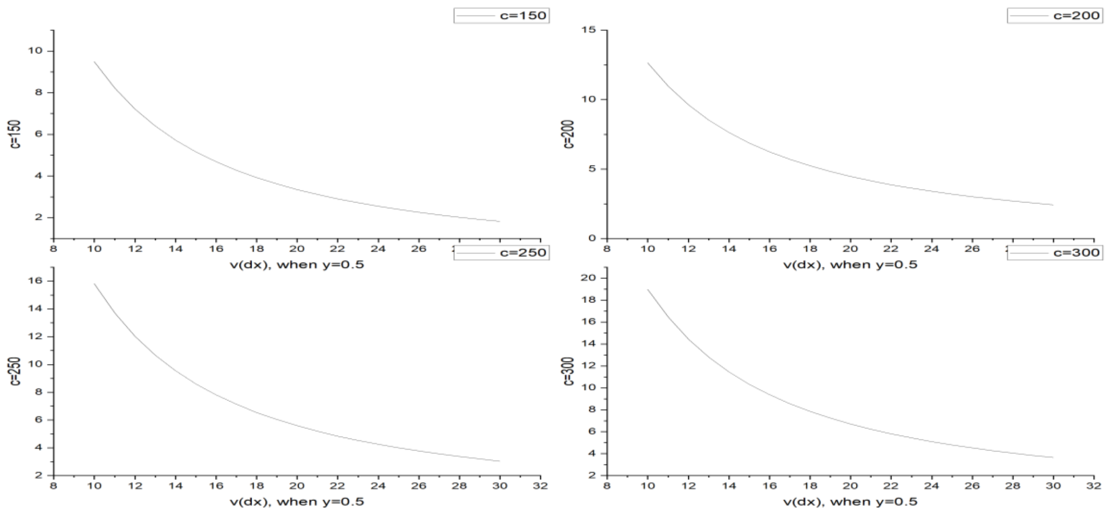

Fig. 3(b)

In Table 3, we consider x =10 to 30 percent, parameter c = 150 to 300, and the value of y = 0.5 years. In Figures 3(a) and 3(b), we can see the stability and fluctuation of $\vartheta(dx)$. So we looked, $\vartheta(dx)$ is a greater and same decrease on the value of parameters c = 150, c = 250, c=300 and a lesser decrease in the parameter c =200 value.And it is showing positive half tail.

Table 4

x	c=150, y=1	c=200, y=1	c=250, y=1	c=300, y=1
10	3	4	5	6
11	2.479339	3.305785	4.132231	4.958678
12	2.083333	2.777778	3.472222	4.166667
13	1.775148	2.366864	2.95858	3.550296
14	1.530612	2.040816	2.55102	3.061224
15	1.333333	1.777778	2.222222	2.666667
16	1.171875	1.5625	1.953125	2.34375
17	1.038062	1.384083	1.730104	2.076125
18	0.925926	1.234568	1.54321	1.851852
19	0.831025	1.108033	1.385042	1.66205
20	0.75	1	1.25	1.5
21	0.680272	0.907029	1.133787	1.360544
22	0.619835	0.826446	1.033058	1.239669
23	0.567108	0.756144	0.94518	1.134216
24	0.520833	0.694444	0.868056	1.041667
25	0.48	0.64	0.8	0.96
26	0.443787	0.591716	0.739645	0.887574
27	0.411523	0.548697	0.685871	0.823045
28	0.382653	0.510204	0.637755	0.765306
29	0.356718	0.475624	0.59453	0.713436
30	0.333333	0.444444	0.555556	0.666667

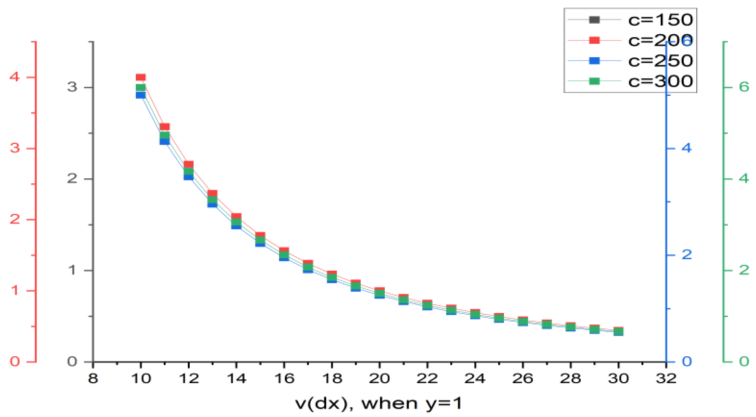

Fig. 4(a)

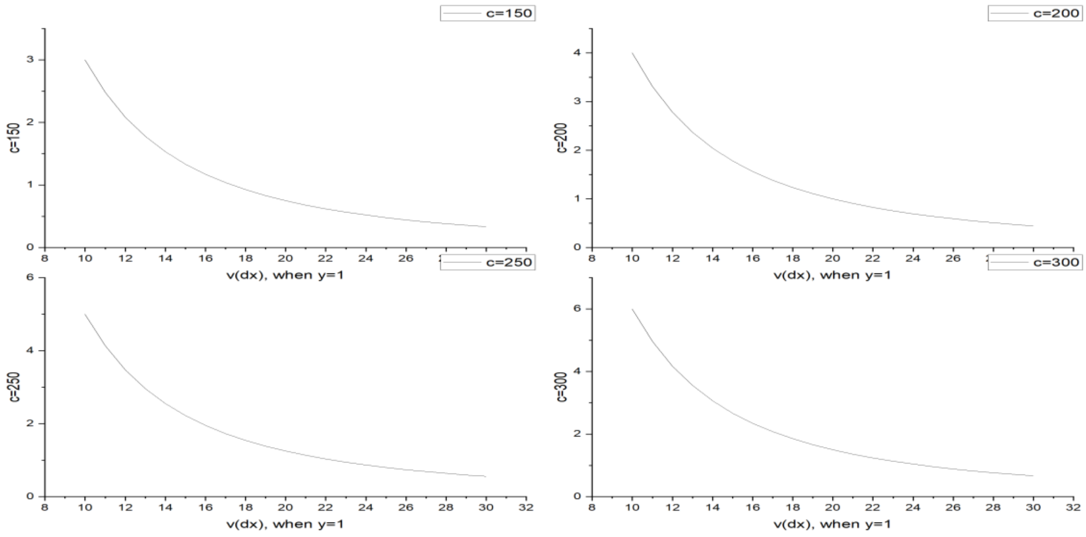

Fig. 4(b)

In Table 4, we consider x =10 to 30 percent, parameter c = 150 to 300, and the value of y = 1 years. In Figures 4(a) and 4(b), we can see the stability and fluctuation of $\vartheta(dx)$. So we looked $\vartheta(dx)$isdecrease in the parameters c = 150, c =200, c =250, and 300 and met at the same point until x = 30.And it is showing positive half tail.

Table 5

x	c=150, y=1.5	c=200, y=1.5	c=250, y=1.5	c=300, y=1.5
10	0.948683	1.264911	1.581139	1.897367
11	0.747549	0.996732	1.245915	1.495098
12	0.601407	0.801875	1.002344	1.202813
13	0.492337	0.65645	0.820562	0.984675
14	0.409073	0.545431	0.681789	0.818147
15	0.344265	0.45902	0.573775	0.68853
16	0.292969	0.390625	0.488281	0.585938
17	0.251767	0.335689	0.419612	0.503534
18	0.218243	0.29099	0.363738	0.436486
19	0.19065	0.2542	0.31775	0.3813
20	0.167705	0.223607	0.279508	0.33541
21	0.148448	0.19793	0.247413	0.296895
22	0.132149	0.176199	0.220249	0.264298
23	0.11825	0.157667	0.197084	0.2365
24	0.106315	0.141753	0.177191	0.212629
25	0.096	0.128	0.16	0.192
26	0.087034	0.116045	0.145056	0.174068
27	0.079198	0.105597	0.131996	0.158395
28	0.072315	0.09642	0.120524	0.144629
29	0.066241	0.088321	0.110402	0.132482
30	0.060858	0.081144	0.10143	0.121716

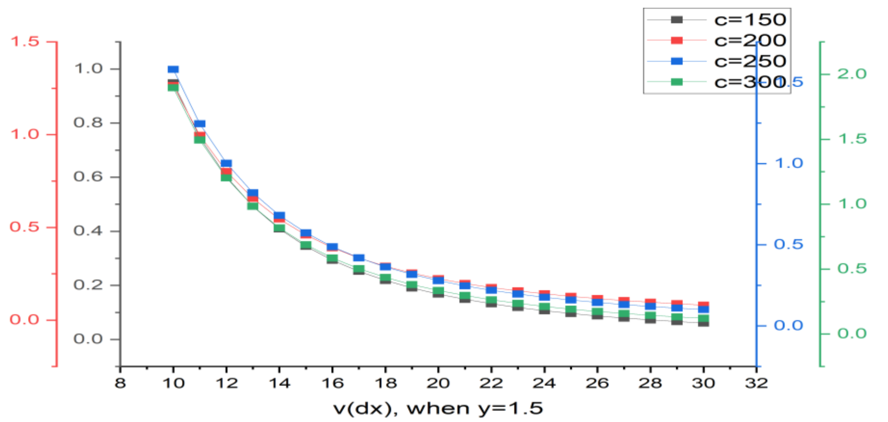

Fig. 5(a)

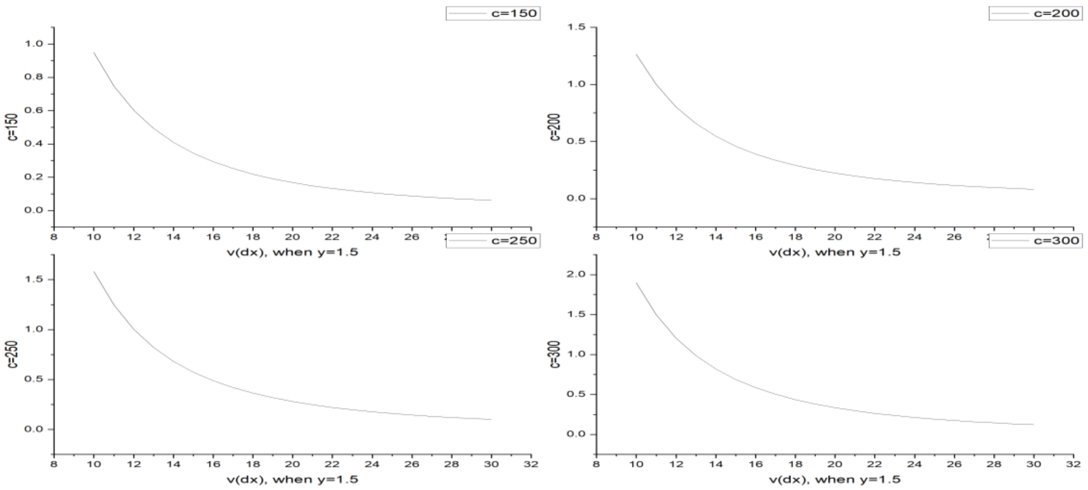

Fig. 5(b)

In Table 5, we consider x =10 to 30 percent, parameter c = 150 to 300, and the value of y = 1.5 years. In Figures 5(a) and 5(b), we can see the stability and fluctuation of $\vartheta(dx)$. So we looked $\vartheta(dx)$ is equal and less decrease on the parameters c =200, c =250, and 300. More decrease in the parameter c =150 value. And it is showing positive half tail.

CONCLUSION

With the homotopy perturbation approach, the Adomian decomposition method, and the variational iteration method, it might be possible to get good analytic-numerical approximations for the probability density function of univariate stable distribution. For the purpose of modeling heavy tail models, stable distributions, including tempered stable distributions, might prove helpful. As an accurate governing mechanism, fractional calculus is heavily relied on in the fields of continuum mechanics, elasticity, signal analysis, quantum mechanics, bioengineering, biomedicine, financial systems, social systems, pollution, control, turbulence, population growth and dispersal, landscape evolution, medical imaging, and complex systems.

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