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Mathematical Reasoning Error Analysis of College Students for a Proposed Plan of Action

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Abstract
Mathematical reasoning skills play an important role in student academic success. This study aimed to explore the mathematical reasoning ability of college students. The study utilized explanatory sequential mixed method research design to explore the level of mathematical reasoning ability of 30 college students who were selected using random sampling. Researcher made test was used to gather quantitative data while for the qualitative data, an in depth interview following Newman Interview Prompt was utilized. Data revealed that the level of mathematical reasoning of college students in terms of analyzing is on developing, while forming conjectures and generalizing and justifying can be both describe as on the beginning level. The common errors committed by the respondents based on the qualitative data were misinterpreting data, calculation errors, and ignoring context during analyzing. Moreover, it was also revealed that during forming conjectures and generalizations, students tend to commit over generalization and inappropriate use of patterns. Finally, for justifying and logical arguments, errors were lack of justifications, logical fallacies, incomplete proofs and misunderstanding definition. It was also revealed that common cause of errors in mathematical reasoning were confusion in algebra, confusion in communication, confusion on notation, unjustified generalization and misconception in reasoning. Action was proposed which focuses on increasing college student mathematical reasoning was proposed.

Keywords
Mathematical Reasoning, Error Analysis, Intervention Plan

INTRODUCTION

Mathematics has great value in various facet of life, and this is apparent through its multiple applications in all scientific and life fields, as it is characterized by keeping pace with the improvement in every time and place. Given this value, countries are keen to prioritize the teaching and learning of Mathematics for their students by selecting the foremost methods and techniques.

Essential mathematical abilities, such as reasoning, should be taught to kids as early as those in the primary schools. Analyzing and assessing Mathematical claims or assertions without taking into account their context or meaning is the crucial ability of mathematical reasoning (Mueller et al., 2014). Students can use this Mathematical idea to ascertain the veracity of statements that are presented. People may solve arithmetic issues using mathematical reasoning without the need for algorithms or preset procedures. In order to draw connections and identify the optimal solution, it comprises applying critical and logical thinking to a mathematical issue (Curriculum Development Division, 2019).

To comprehend Mathematics more efficiently and meaningfully, mathematical reasoning is a must. Students' intellectual and communicative development is intimately linked to their ability to think mathematically. The ability to reason logically and even more critically is a skill that reasoning can help one develops. Critical thinking is the cornerstone of a deep and meaningful comprehension of mathematics (Payadnya, 2019). In order to give students the space and chance they need, teachers must create lessons and learning activities that compel them to do mathematical operations and actively participate in mathematical discussion.To guarantee that students' thinking may be enhanced, teachers must implement mathematical reasoning as a crucial ability in their Mathematics teaching activities (Morsanyi et al., 2018). Students who have mastered reasoning skills are able to comprehend abstract mathematical topics.

Additionally, it helps students solve a variety of mathematical puzzles using a variety of inventive and logical approaches, especially non-routine mathematical puzzles. In addition to providing a more meaningful grasp of the subject of mathematics, mathematical reasoning is a fundamental building block for students to comprehend mathematics more effectively (Mahmud et al., 2020).

It is where analyzing inaccurate worked examples in order to help students gain a conceptual knowledge of mathematical concepts is of importance. Students' procedural knowledge can be enhanced by analyzing their own mistakes, which can also help them recognize when they have selected the wrong course of action. Error analysis is a common method used when students make mistakes on a regular basis to identify the source of their errors. It entails reviewing a student's work to look for any instances where there might be miscommunication. Factual, procedural, and conceptual errors in mathematics can occur for a number of reasons. More significantly, error analysis supports students in correcting their understandings by assisting them in identifying their misconceptions.

The problem is that, according to the PISA 2018 International Report, the average mathematical literacy score of Filipino pupils was far lower than the average of the Organization for Economic Cooperation and Development (OECD), suggesting a level of skill below Level 1 (OECD, 2019). Concerns about students' reasoning aptitude levels and their inadequacies in applying reasoning to mathematics learning activities and solving a range of non-routine problems must be taken into consideration. The researcher became aware of this problem as she personally observed this scenario. As a Mathematics instructor in the tertiary level, she witnessed how college students deal with this problem.

In an attempt to amend students’ reasoning skills in mathematics,she became interested in exploring the reasoning skills of college students, thus identify different errors committed and used it as a framework for a propose intervention plan to improve their mathematical reasoning skills.

MATERIALS AND METHODS

The researcher used a mixed method research approach for this investigation. An strategy of investigation known as "mixed method research" associates or mixes both qualitative and quantitative elements. Researchers from a variety of academic fields can address research topics with rigor when using mixed method design. In particular, an explanatory sequential mixed method research design was used by the investigator. The design is two-phase, with the collection and analysis of quantitative data coming first, followed by the collection and analysis of qualitative data in light of the quantitative findings. The quantitative facts are explained by the qualitative information. In this particular study, the respondents' mathematical reasoning was characterized numerically, but during the in-depth interview, the causes of inaccuracy were detailed qualitatively.

The respondents of the study were the 35 college students of Laguna State Polytechnic University, San Pablo City Campus enrolled in GEC 104 for the Second Semester of academic year 2023-2024.

Purposive sampling was used in the study to choose 35 individuals from the researcher's student body enrolled in GEC 104 at LSPU, San Pablo City. The respondents have to meet the requirement of being enrolled in the current semester of Mathematics in the Modern World. They were chosen for the study because they are among the students included in the latest PISA findings, which place the Philippines bottom in the world for mathematical reasoning.

The questionnaire used in the study was divided into four parts. The first part gathered the profile of the respondents as to age, sex and the college they belong to. The second part is a researcher-made test which assessed the level of mathematical reasoning ability of the respondents as to analyzing, forming conjecturing and generalizing and logical arguments. The third part is another researcher-made test which identified the errors committed by the respondents. The last part is an adopted interview prompts from Newman which was used in an in-depth interview to further assess the common errors committed by the respondents.

All questionnaires were subjected to validation by experts on the field of mathematics and research before proceeding to the actual phase of data gathering of the study.

Upon obtaining approval for the study, the researcher sought permission from the Campus Director of LSPU, San Pablo City. Once permission has been secured, the study progressed to the next phase. Questionnaires were distributed in a face-to-face manner, ensuring privacy and confidentiality during both distribution and retrieval. The collected data were then checked and given corresponding scores. Also, the answers submitted by the learners were assessed using a rubric adopted from ACARA (2017) to evaluate the mathematical reasoning of the respondents. These include analyzing, forming conjectures, generalizing, and justifying and logical arguments. Subsequently, the researcher selected three students from each category—high, average, and below-average scores—for interviews. The interviews followed the steps of error analysis outlined by Howell, Fox, & Morehead (1993). This analysis aimed to systematically identify errors committed by each student and discern the underlying causes. The interview questions covered errors that might be committed during different stages in reading, comprehension, transformation, process skills, and encoding. Students were asked to think aloud or speak during the interview while they solved the issues without the researcher giving them any hints or prompts. Initially, the researcher collected samples of student work for each type of problem. Responses from the students were noted. Patterns were found once it was analyzed. Later, the pupils were given a description and discussion of it. The results of the error analysis were utilized as a foundation for constructing a framework for an intervention plan. This plan was designed to enhance the mathematical reasoning skills of the respondents based on the identified errors. Examples of students' reasoning aligned with elements and levels guided the development of this intervention plan. The instructional interventions developed through this process aimed to enhance the analysis of mathematical reasoning errors among college students. This intervention plan served as the basis for creating self-paced learning modules, providing a comprehensive approach to improving mathematical reasoning skills.

In assessing the level of reasoning skills and errors committed by the respondents' descriptive statistics such as frequency and percentage were used. For the qualitative data, interview data were transcribed verbatim and manually. They were coded, interpreted and subjected to thematic analysis to identify common causes of errors from each of the respondents.

RESULTS AND DISCUSSIONS

Level of Mathematical Reasoning of College Students

Table 1 presents the level of mathematical reasoning of college students. Data suggest that in a Mathematical reasoning assessment focusing on Analyzing as a parameter, it is noteworthy that the majority of students are operating at the Developing level, with scores ranging between 11 and 15. This level of proficiency indicates that students demonstrated the ability to recognize common numerical or spatial properties within algebraic sets and functions problems. They were capable of noticing patterns and structures, recalling them, and extending them within the context of mathematical reasoning. Moreover, students at this level exhibited competency in sorting and classifying cases based on common properties, showcasing a foundational understanding of algebraic concepts. However, their abilities may still be limited to relatively straightforward applications, and they may require further development to tackle more complex problem-solving scenarios. As identified by the Australian Curriculum Assessment and Reporting Authority, (2015) students who master analyzing should be able to investigate problems using given examples to verify a hypothesis. They can identify similarities and differences, arrange and categorize examples. Moreover, they can identify what remains and what evolves as well as to replicate or broaded pattern. But unfortunately, within algebra sets and functions work problems, students at the Developing level have limited skills at identifying recurring patterns and properties. They exhibit the ability to recall and repeat these patterns, utilizing numerical or spatial structures to make sense of the problem at hand. Additionally, their capacity to sort and classify cases according to shared properties reflects a burgeoning comprehension of algebraic concepts. However, while they can recognize and apply these foundational principles, their skills may not yet extend to analyzing more intricate algebraic relationships or solving advanced problems requiring abstract reasoning. Therefore, while students at this level demonstrate proficiency in basic algebraic reasoning, there remains room for growth and further development to advance to higher levels of mathematical proficiency. As what ACARA (2015) claimed, analyzing is vital for students and consider it as one that must be maintained and taught to students.

Table 1 Level of Mathematical Reasoning of College Students

Scores	Analyzing		Forming Conjectures and Generalizing		Justifying and Logical argument		Interpretation
	f	%	f	%	f	%	
21-25	-	-	-	-	-	-	Extending
16-20	5	14.3	1	2.9	-	-	Consolidating
11-15	18	51.4	10	28.6	7	20.0	Developing
6-10	12	34.3	24	68.6	27	77.1	Beginning
0-5	-	-	-		1	2.9	Not Evident
Total	**35**	**100.0**	**35**	**100.0**	**35**	**100.0**	

On the other hand, in a math reasoning assessment focusing on Forming Conjectures and Generalizing as a parameter, majority of students are situated at the Beginning level, with scores ranging from 6 to 10. This level of performance signifies that students primarily rely on basic methods such as drawing and counting to identify and communicate singular common properties or repeated components within patterns. Their ability to use diagrams or materials to add to existing patterns demonstrates an initial understanding of pattern recognition and manipulation. However, students at this level are still in the early stages of developing the capacity for conjecture formation and generalization, as their skills are primarily centered around concrete representations and simple observations only. According to Australian Curriculum Assessment and Reporting Authority, (2015), students who master forming conjectures and generalization usually able to formulate claims even without verifying it and able to common characteristics or patterns in multiple cases and expressing a rule (conjecture) to characterize the shared characteristic, pattern, or relationship. Within the realm of pattern recognition and manipulation, students at the Beginning level showed promise in utilizing visual aids such as drawings or physical objects to identify common properties or components within patterns. Their proficiency in adding to existing patterns suggests a foundational grasp of pattern extension and modification. Nevertheless, their ability to form conjectures and generalize based on these patterns remains limited, as they may struggle to extend their observations beyond concrete examples or to articulate broader mathematical principles. Thus, while students at this level exhibit potential for growth, they require further guidance and instruction to progress towards more sophisticated levels of mathematical reasoning and conjecture formation. This is a need that the teachers have to attend to. Teachers play a vital role in the development of such mathematical reasoning ability. The potential of the task to encourage reasoning, appropriate actions to explore reasoning with students, and comprehension of the work produced by the students in order to support them in the most appropriate way are just a few of the factors that teachers must be aware of in order to foster reasoning ability in their students, as highlighted by Davidson et al. (2019).

In terms of math reasoning assessment emphasizing Justifying and Logical argument as a parameter reveals that the majority of students fell within the Beginning level, scoring between 6 and 10. At this stage, students demonstrated an ability to describe their actions and offer explanations for their correctness or potential errors in solving algebra sets and functions work problems. They are proficient in recognizing correct or incorrect solutions using various tools such as materials, objects, or verbal reasoning. However, their judgments are primarily based on simple criteria, often relying on known facts or basic rules without delving deeply into the underlying reasoning processes. Consequently, while students at this level exhibit some capacity for logical thinking and justification, their arguments may lack coherence and may not encompass all necessary steps in the reasoning process, indicating a need for further development in constructing rigorous mathematical arguments. Within algebra sets and functions work problems, students at the Beginning level typically employ straightforward reasoning strategies to evaluate solutions. They were capable of identifying correct and incorrect approaches, drawing on their understanding of basic mathematical principles. However, their judgments may be constrained by a reliance on surface-level observations rather than a deep analysis of the underlying logic. Consequently, their arguments may lack depth and coherence, and they may overlook critical steps in the reasoning process. Such qualities were still far from the characteristics of students who master justifying and forming logical arguments based from the Australian Curriculum Assessment and Reporting Authority (2015) which claimed that students should be able to either support or contradict a claim and persuade others of its veracity through utilizing concepts that are already known, adhering to established procedures or stages when presenting argument and to make use of well-known and understood terminology, diagrams, and symbols. To progress beyond the Beginning level in justifying and constructing logical arguments, students need to develop a more comprehensive understanding of mathematical concepts and cultivate the ability to articulate coherent reasoning chains that encompass all relevant steps in problem-solving processes.

Findings with regards to students mathematical reasoning ability is quite disturbing as student failed to showcase higher mathematical reasoning ability. This is attributed to poor instructions during the Covid19 pandemic where students were forced to study on their own without proper guidance of teachers. This findings was confirmed by Ramadhany (2021) who claimed Covid19 Pandemic influences the development of mathematical reasoning ability of students. Findings suggest that there was a decline in students mathematical reasoning ability and only 14.29% of students have strong mathematical thinking skills and can execute each and every reasoning indicator.

Common Errors in Mathematical Reasoning

Mathematical Reasoning is an ability that allows students to apply critical thinking in mathematics. It is a concept that educators use in the classroom to help students grasp this subject better. It helps pupils develop a pattern of thought that facilitates answering challenging issues. Unfortunately, development of mathematical reasoning is being hindered by different errors a student commits. Table 4, presents the common errors committed by college students.

Table 2 Common Errors Student Respondents Committed

Common Errors	Description	Number of Cases
1. Analyzing	*1.1. Misinterpreting Data:* Students may misread graphs, tables, or word problems.	18 / 35
	1.2. Calculation Errors: Incorrect arithmetic, algebraic manipulations, or improper use of formulas.	14 / 35
	1.3. Ignoring Context: Overlooking units, scales, or the context of a problem.	12 / 35
2. Forming Conjectures and Generalizing	*2.1. Overgeneralization:* Making broad statements that are not supported by sufficient examples.	13 / 35
	2.2. Inappropriate Use of Patterns: Mistaking coincidental patterns for general rules.	18 / 35
3. Justifying and Logical Argument	*3.1. Lack of Justification:* Failing to provide or did not provide reasoning or evidence for their conjectures.	15 / 35
	3.2. Logical Fallacies: Using invalid reasoning, such as circular reasoning or false dilemmas.	15 / 35
	3.3. Incomplete Proofs: Leaving out crucial steps or failing to cover all cases in a proof.	14 / 35
	3.4. Misunderstanding Definitions: Using terms or concepts incorrectly to justify certain parts of solution.	22 / 35

Table 4 shows the common errors committed by the student respondents on the entire conduct of the study. This provides a first-hand insight on the areas of struggle on the learners in their mathematical reasoning skills. Within the Analyzing category, misinterpreting data is the most frequent error, 18 out of 35 cases. This implies that most of the respondents often have problems understanding and analyzing the word problems. Aside from this, another notable and frequent error is calculation error and ignoring context having frequencies of 14 and 12 out of 35 respectively.

Under Forming Conjectures and Generalizing, the inappropriate use of patterns were the most prevalent error committed by the student respondents, having 18 cases out of 35. This indicates a common issue where students may incorrectly identify patterns and assume general rules based on insufficient evidence. Overgeneralization is another notable error, appearing in 13 out of 35 instances. This reflects a tendency among students to make broad statements without adequate examples, highlighting a gap in their ability to critically assess the validity of their conjectures.

Lastly, under "Justifying and Logical Argument" category, misunderstanding definitions and concepts stand out as the most frequent error across all 3 main categories, having 22 out of 35 instances of committed error. This suggests that students often misuse terms or concepts, which undermines their ability to justify their solutions accurately. Additionally, lack of justification and logical fallacies are each observed in 15 cases, while incomplete proofs occur in 14 cases. These findings indicate that students frequently fail to provide thorough reasoning and evidence for their solutions, rely on invalid reasoning, and neglect essential steps in their proofs.

Findings was in relation with the study of Nwoke et al. (2024), which revealed that the errors committed by students included conceptual errors, computational errors, defective algorithms, and wrong operational errors. Overall, these patterns emphasize the need for enhanced instruction in critical thinking, precise use of mathematical terminology, and the development of robust logical reasoning skills.

Common Causes of Errors in Mathematical Reasoning

According to Newman (1977) errors can be committed in every stage of problem solving. It can be from reading, comprehension, transformation, processing skills and encoding. According to Sarwadi and Shahrill (2014), such errors may have a significant impact on students' academic success. Moreover, it was also noted that errors can be different from student to students and have varying causes.

Interview from the students who got the lowest score revealed varied causes of errors.

A. Confusion in Algebra

There are different types of errors due to confusion in algebra. From the most common which is sign errors to errors relating to improper operation and breaches of mathematical properties of numbers. Sign errors appear to increase as students approach the middle and end of the year and are working with increasingly complex equations. However, students who struggled with arithmetic at the beginning of the year are the ones who are still learning how to solve basic equations. This may imply that sign errors, which arise when there are several variables in an equation to handle, are typical and indicative of an average learner. However, making mistakes with variables when working with a single variable can suggest that a student lacks a firm understanding of variables, which will limit their ability to move further. This suggests that ensuring that students have a firm grasp of the idea of variables, specifically their comprehension of which terms are "like" and which are "unlike," would be an excellent candidate for intervention. For instance, the respondents said,

"*I don't have it on paper (unknown and known facts). But at first, I am thinking of the concept of ratio is, but I don't know how to apply it*."

According to Booth et al. (2014), some error types are suggestive of problems with arithmetic accomplishment at particular content levels, and different error types are more common with different content.

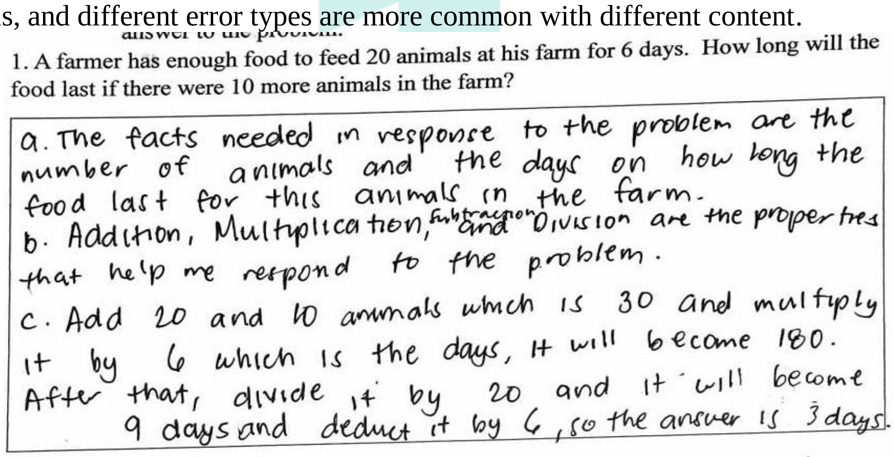


Fig. 1 Sample Error Caused by Confusion on Algebra

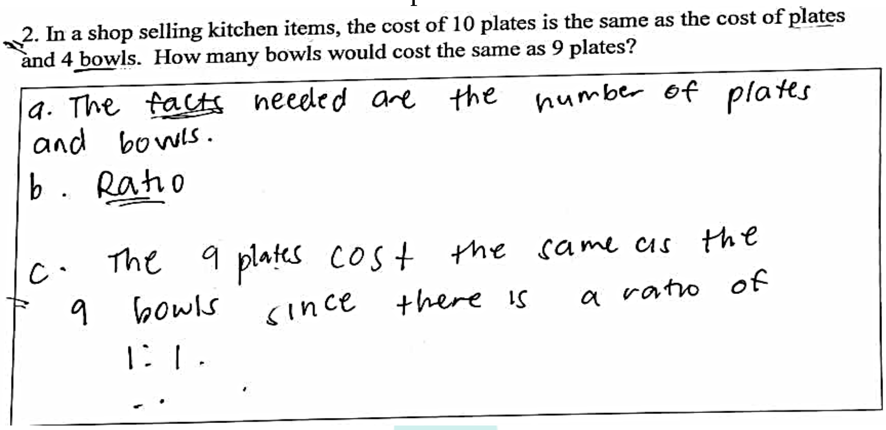
Figure 1 shows the sample error caused by confusion in algebra which focuses on variable error. It was evident that the student recalls some information from the question but the answer is not clear, the situation is misinterpreted and did not correspond to correct Mathematical sentence.

B. Error Caused by Confusion in Communication

One of the most important aspects of solving mathematical problems is mathematical communication. The National Council of Teachers of Mathematics (NCTM) has stated that mathematical communication is one of the fundamental skills children have for solving mathematical issues, which clarifies the importance of mathematical communication in answering problems rapidly (NCTM, 2020). The NCTM lists four requirements for mathematical communication in math education. Students must be able to: (1) organize and consolidate their mathematical thinking through communication; (2) communicate their mathematical thinking in a coherent and clear manner to peers, teachers, and others; (3) analyze and evaluate the mathematical ideas and strategies of others; and (4) use mathematical language in order to present mathematical ideas in an appropriate manner.

When students have strong mathematical communication skills, they can clearly explain concepts and solutions for solving mathematical problems (Qohar, 2011). When students solve mathematical problems and effectively communicate their ideas, teachers are better able to spot misconceptions or procedural errors that students may be making (Wilson, 2019). When students are able to interpret the meaning behind their responses, even in cases when the mathematical concepts they transmit are erroneous, the teacher can still detect the errors made by the pupils. Conversely, if students possess intelligence but lack excellent communication skills, it will be difficult for the teacher to discern between the accurate and inaccurate mathematical notions that the students are expressing (Cristobal and Lasaten, 2018).Good communicators of ideas will indirectly influence the teacher's assessment of the students' answers. The right mathematical concept will be misunderstood if someone reads or evaluates a student's response improperly if it is not expressed clearly (Ramadhany and Dwi, 2018). Thus, the ability of students to communicate mathematical ideas is essential. NCTM made it clear that communication can help pupils grasp mathematics thoroughly (NCTM, 2020). Students who solve mathematical problems or use mathematical reasoning effectively might also become more adept at persuading others of their points of view through good communication. Students who listen to their peers discuss mathematics will learn more and advance in their mathematical reasoning. Students should be able to approach difficulties methodically, as this will help them later on in the community when they communicate effectively (Umar, 2012). Unfortunately it was evident that the respondents were still not able to master and thus committed mistake because of communication error. The respondent said "*I think it is ratio. What I understand is that 4 plates is equal to 4 bowls. It is 1:1. So I thought that the price of 1 plate is equal to 1 bowl. And the question asked if how many bowls has the same value as 9 plates so my answer is 9 bowls*."

According to a recent study by Maulyda et al. (2020), students are still having trouble explaining their mathematical ideas clearly. One common mistake students make when writing is converting difficult sentences into mathematical models. Students continue to solve writing problems in unorderly and illogical manner. The study's findings demonstrated that pupils' oral communication skills are superior to their written communication skills.


2. In a shop selling kitchen items, the cost of 10 plates is the same as the cost of plates and 4 bowls. How many bowls would cost the same as 9 plates?

a. The facts needed are the number of plates and bowls.

b. Ratio

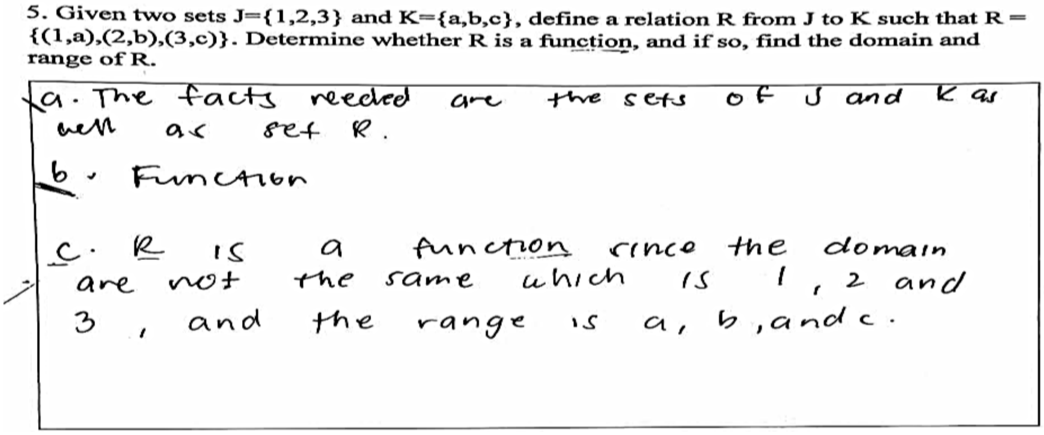
c. The 9 plates cost the same as the 9 bowls since there is a ratio of 1:1.

Fig. 2 Sample Error Caused by Confusion in Communication

Figure 2 shows the sample error caused by confusion in communication committed by the respondents. The student was not able to communicate the intended property to use. The idea of a ratio is not enough to equate, mistranslated and communicate the answer.

C. Confusion on Notation

Under notation confusion, Schecter (2009) distinguished between many types of errors. This includes idiosyncratic inverses, confusion with the square root symbol, issues with the order of operations, imprecisely expressed fractions, and equalities and implications from stream of consciousness were all types of confusion on notation. Issues with order of operation was the most common cause of student error under confusion on notation. Evident on statement such as "*I know how to identify a function but I cannot state clearly the properties formally using symbols.*"


5. Given two sets J={1,2,3} and K={a,b,c}, define a relation R from J to K such that R = {(1,a),(2,b),(3,c)}. Determine whether R is a function, and if so, find the domain and range of R.

a. The facts needed are the sets of J and K as well as set R.

b. Function

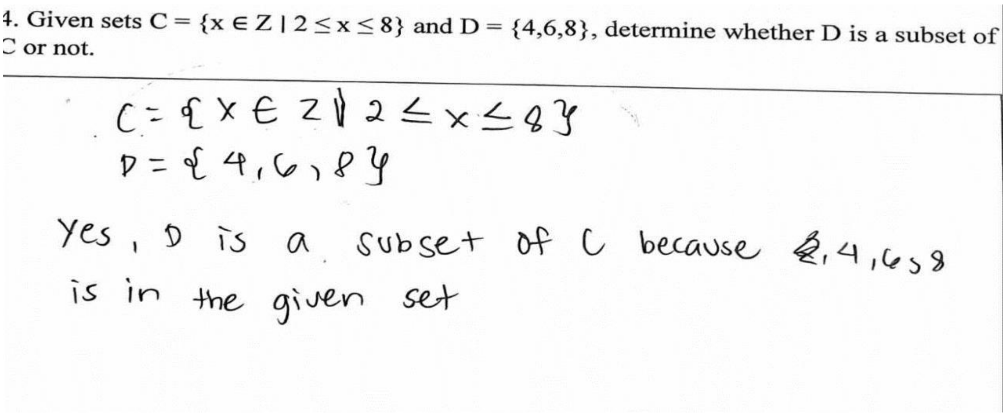
c. R is a function since the domain are not the same which is 1, 2 and 3, and the range is a, b, and c.

Fig. 3 Sample Error caused by Confusion on Notation

Figure 3 portrays the sample error caused by confusion on notation. The student tried to analyze and present the concept of relations and functions but no clear explanations were given. Item was generalized using single statement but it is not clearly enough as to how he arrived at the given answer. No justifications were also given.

D. Unjustified Generalization

According to Schechter (2009), unjustified generalization occurs when students attempt to apply a formula or notation that may be appropriate in one context to a larger one when it may not be appropriate at all. Another name for this is "crass formalism." This mistake occurs when we extrapolate large conclusions from small amounts of evidence. This might occur if an unrepresentative sample or an abnormally small sample supports the generalization. It was evident on statement “*I am not sure on what elements are included in sets C.*”


4. Given sets C = {x ∈ Z | 2 ≤ x ≤ 8} and D = {4,6,8}, determine whether D is a subset of C or not.

$C = \{x \in Z \mid 2 \leq x \leq 8\}$

$D = \{4,6,8\}$

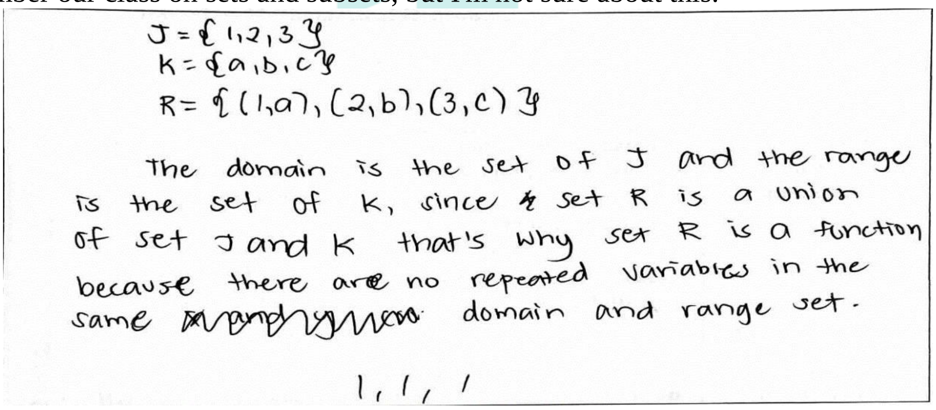
Yes, D is a subset of C because ~~2~~,4,6,8 is in the given set

Fig. 4 Sample Error Caused by Unjustified Generalization

Figure 4 portrays the sample error caused by unjustified generalization, the student tried to analyze data but did not manage to identify properties or concept under relations and functions. He failed to communicate any common property or rule and tried to give false justifications on determining functions.

E. Error Caused by Misconception in Reasoning

According to Schechter (2009), there are a number of common examples of this error, such as going over work again, working backward, not realizing that some steps are irreversible, mixing up a statement and its opposite, struggling with quantifiers, employing the wrong approach but supported by one or two instances of accurate results, and putting too much trust in calculators. It is clear from the sentence, "I'm not entirely sure how to distinguish between a domain and a range. I think I remember our class on sets and subsets, but I'm not sure about this.


$J = \{1,2,3\}$

$K = \{a,b,c\}$

$R = \{(1,a),(2,b),(3,c)\}$

The domain is the set of J and the range is the set of K, since set R is a union of set J and K that's why set R is a function because there are no repeated variables in the same domain and range set.

1, 1, 1

Fig. 5 Sample Error Caused by Misconception in Reasoning

Figure 5 presents the sample error caused by misconception in reasoning. It was evident that students correctly analyzed the set into a relation but no clear explanation as to property used was given. The student applied the same concept to all the items resulting to applying the wrong concepts on functions.

CONCLUSIONS

The level of mathematical reasoning of college students in terms of analyzing is on developing, while forming conjectures and generalizing and justifying can be both describe as on the beginning level.The common errors committed by the respondents based on the qualitative data were misinterpreting data, calculation errors, and ignoring context during analyzing. Moreover, it was also revealed that during forming conjectures and generalizations, students tend to commit over generalization and inappropriate use of patterns. Finally, for justifying and logical arguments, errors were lack of justifications, logical fallacies, incomplete proofs and misunderstanding definition. The common cause of errors in mathematical reasoning were confusion in algebra, confusion in communication, confusion on notation, unjustified generalization and misconception in reasoning. Action plan was proposed which focuses on increasing college student mathematical reasoning was proposed.

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DECLARATION OF CONFLICT
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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